Georg Weinblum Special Meeting

19-20 March 1997

A Special Meeting was held after the Twelfth Workshop to celebrate the 100th anniversary of the birth of Georg Weinblum. Professor Weinblum was an international leader in ship theory. He inspired a generation of colleagues, including several who are still active participants in the Workshops. For this reason it was felt that the anniversary celebration should be held in conjunction with the Twelfth Workshop.

Since Weinblum's death in 1974, a series of Memorial Lectures have been presented on an annual basis (the list is given at the end of this volume). All of the former Lecturers were invited to participate in the Special Meeting, and to present lectures. Thirteen among them contributed. Titles of their presentations and short written abstracts are given in the following pages.

Georg Weinblum

Georg Weinblum Special Meeting

List of contributions

Stagnation Points

K. J. Bai[1], C. W. Dawson[2], J.W.Kim[3], and J. V. Wehausen[3]

A rectilinear potential flow about a circle in the plane or about a sphere in three dimensions results in two stagnation points, one at each end of a diameter. For any bounded simply connected region in the plane it follows from Riemann's Mapping Theorem that there is an analytic function mapping the exterior of the unit circle into the exterior of the region and behaving like a rectilinear flow at infinity. Hence there are only two stagnation points on the boundary of the region in question. Although harmonic functions in three dimensions share many properties with analytic functions in the plane, there is no analogue of the Riemann theorem. In fact, there is only a very restricted set of transformations that preserve the property of being harmonic (see, e.g., Kellogg, *Foundations of Potential Theory* , 1929, pp. 235-236). It is natural to ask whether there can be more than two stagnation points in a potential flow about a bounded simply connected body in three dimensions. The question is raised in Kellogg (ibid., pp. 273-277) but not really answered. It *is* shown in Kellogg (p. 273) that there cannot be a continuous surface distribution of stagnation points (unless, of course, the potential function is constant). On the other hand, one knows that there can be continuous linear distributions of stagnation points if Laplace's equation can be separated in a particular coordinate system, as in $\Phi(x,y,z) = \varphi(x,y)Z(z)$ with $Z(z) = \text{const.}$ or $\Phi(r, \theta, z) = R(r)\Theta(\theta)Z(z)$ with $\Theta = \text{const.}$ One might be led to conjecture that any continuous line of stagnation points must be associated with a coordinate system in which Laplace's equation may be separated. However, the following is a counterexample (JWK) to this conjecture:

$$\Phi(x,y,z) = (1/2)x^2 y^2 - (1/2)(x^2 + y^2)z^2 + (1/6)z^4,$$

for both the x-axis and the y-axis are lines of stagnation points.

A discussion by one of us (JVW) with Charles Dawson in June 1978 concerning the possibilities of multiple stagnation points resulted in a letter from him dated 28 June 1978 describing his investigation of a 3-dimensional body generated by two dipoles of equal moment situated on a line perpendicular to an oncoming steady rectilinear flow. As is well known, when the separation of the dipoles is zero, one streamline will generate a sphere with stagnation points at opposite ends of a diameter. Dawson correctly predicts the dipole separation at which each of the two stagnation points will begin to separate into three stagnation points, and also the

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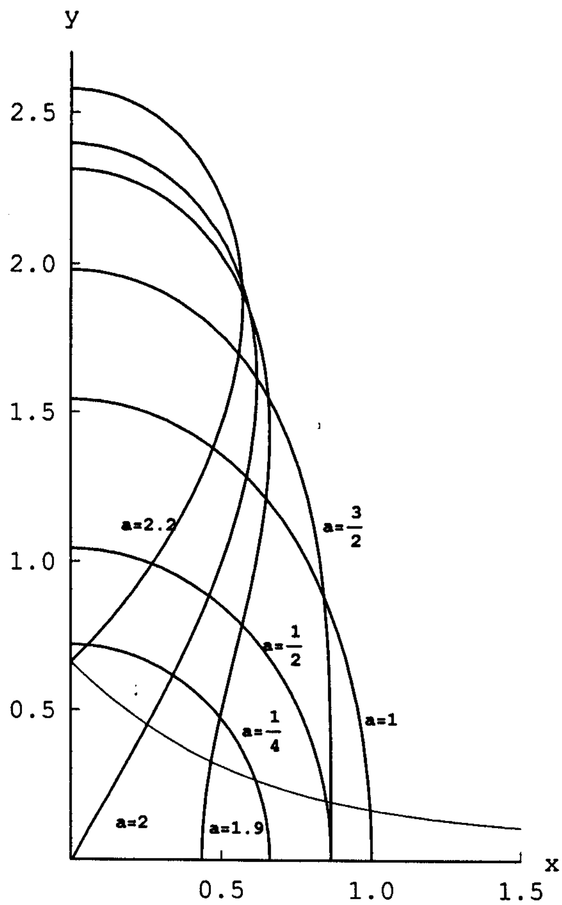
[1] Seoul National University, [2] Formerly, David Taylor Model Basin,
[3] University of California at Berkeley

(further) separation at which the single body will divide into two bodies. In addition, he computed the positions of the stagnation points lying on the central streamline as long as there is only one body.

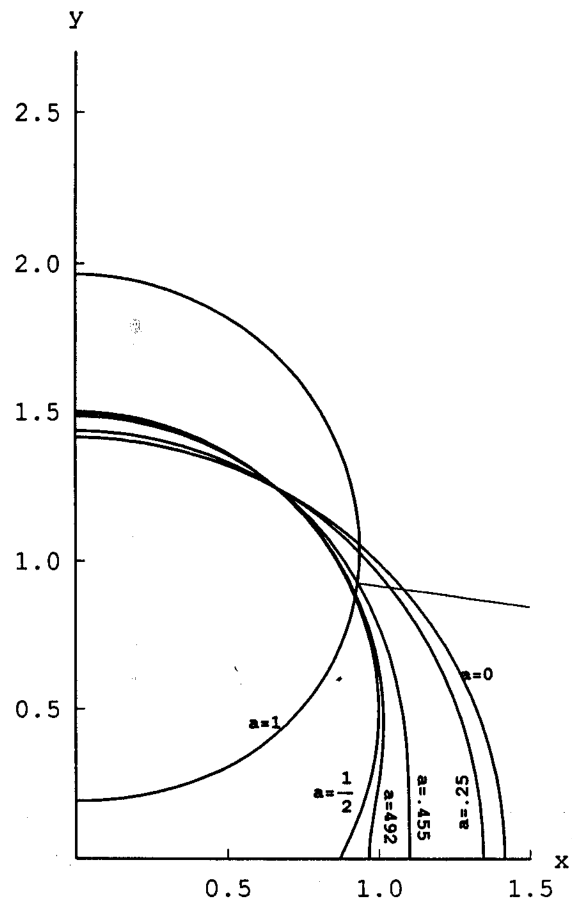
Dawson died in January 1980 without having published any details concerning his calculations. In the present paper we present not only the analysis and computation necessary to substantiate Dawson's results, but also other relevant details accessible by exploiting modern computational capabilities, especially *Mathematica.* In addition to the 3-dimensional problem, we also treat the analogous 2-dimensional problems for two dipoles and for two vortices. The analysis and the computation for these are simpler than for three dimensions, in particular, because of the presence of a stream function, but the results are relevant both for their similarities to and their differences from the 3-dimensional case.

The qualitative difference between two and three dimensions is chiefly a result of the following facts. Let Φ be the velocity potential, in either two or three dimensions, of the rectilinear flow in direction Ox about two dipoles at a distance $2a$ apart and perpendicular to the oncoming flow. In two dimensions Φ_{xx} and Φ_{yy} vanish together at the two stagnation points associated with the largest separation a before a single closed stream body splits into two bodies, with, of course, two stagnation points on each. In three dimensions, however, $\Phi_{yy} = 0$ at a stagnation point associated with a *smaller* value of a than that at which $\Phi_{xx} = 0$, which again occurs at the largest value of a before the single closed stream body divides into two bodies. It is shown, however, that $\Phi_{yy} > 0$ is associated with the presence of two further stagnation points with $y \neq 0$, so that there exists an interval of dipole separations for which there is only one closed stream body but three stagnation points on each side. Furthermore, there exists an interval of separations for which $\Phi_{yy} \geq \Phi_{xx} > 0$, and this implies that the single body is not smooth at the waist, i. e. at the intersection of the stream body with the plane perpendicular to and bisecting the line joining the two dipoles. In two dimensions this nonsmooth behavior can occur only at the "last" single body when $\Phi_{xx} = \phi_{yy} = 0$, the only separation at which Φ_{xx} and Φ_{yy} are equal.

The following two pages show graphs illustrating the differing behaviors for different separations, both for two-dimensional vortex and dipole pairs and for three-dimensional dipole pairs.

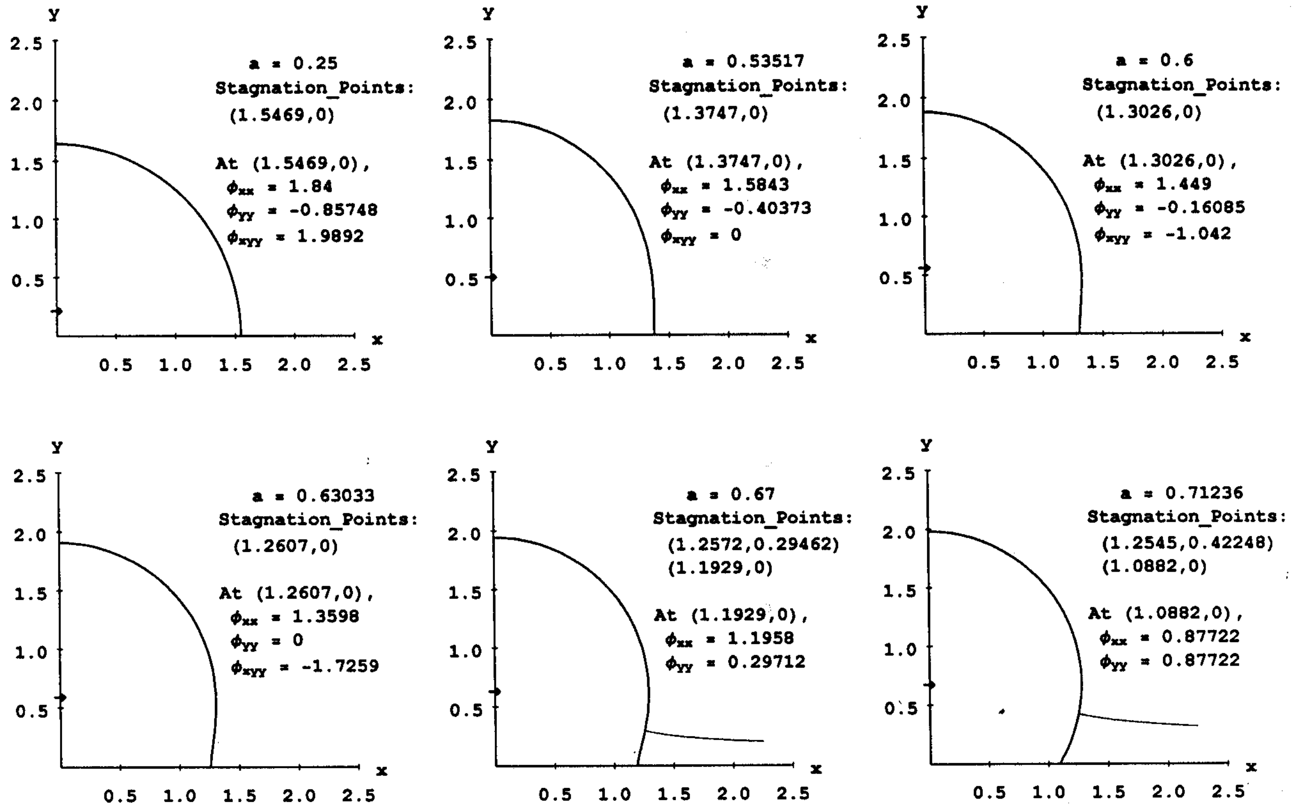


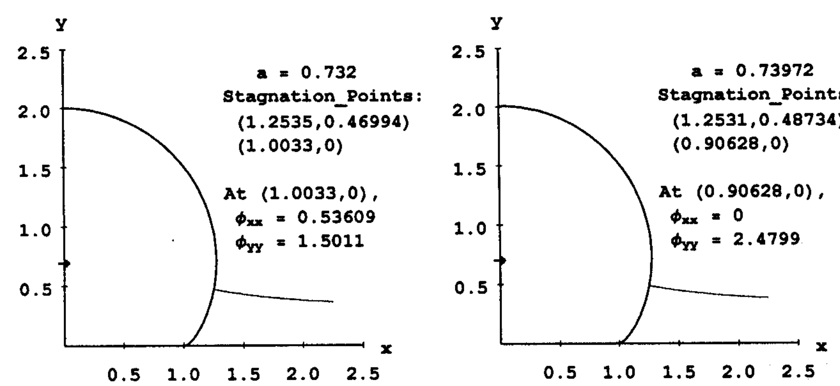
Two vortices in the plane



Two dipoles in the plane

Traces in the first quadrant are shown for the closed streamlines generated by two vortices (on the left), all of the same strength, and by two dipoles (on the right), all of the same moment, but at different spacings, indicated in each case by the value of a. For the vortex pair the largest value of a before two separate bodies are formed is $a = 2$, for the dipole pair this value is $a = 1/2$. In each case this is the value of a associated with $\Phi_{xx} = 0$. The value of a at the boundary between convex and concave behavior at the stagnation point is $a = 3/2$ for the vortices and $a = 0.455 = [(2^{1/2} - 1)/2]^{1/2}$. In each case this is the value of a associated with $\Phi_{xyy} = 0$ at the stagnation point.





Two dipoles in space

Traces in the first quadrant of the (x, y)-plane of the streambodies generated by two dipoles, all of equal moment but with different spacings, as shown by the value of a. All streambodies are bodies of revolution about the y-axis and are symmetric about the (x, z)-plane.

In the first four traces there is a single stagnation point, on the x-axis, in the quadrant shown, and hence one on each side of the streambody. The separation $a = 0.63033$, corresponding to $\Phi_{yy} = 0$, is the largest value of a for which there is only one such stagnation point. The value $a = 0.53517$, corresponding to $\Phi_{xyy} = 0$ at the stagnation point, is the boundary between convex and concave behavior of the streamsurface at the stagnation point. The last four traces, for which $0.63033 < a \leq 0.73972$, all show a second stagnation point at $y > 0$, hence three on each side. At $a = 0.71236$ $\Phi_{xx} = \Phi_{yy}$ at the stagnation point, and the streamsurface has a corner at the (x ,z)-plane. For $a > 0.71236$ this corner becomes an inward-pointing cusp. The largest value of a before the streambody splits into two is $a = 0.73972$.

An Integral Representation of a Wave Function in the Theory of Wave Resistance of Ships

By Masatoshi Bessho

The kernel function of a singularity in the theory of wave resistance of ships is usually represented by a double integral, but it was shown in the memoirs by the author that the kernel function has a single integral representation which faciliates numerical works exeedingly[3].

However, the analysis of the memoirs is somewhat obscure and may have some errors. In fact some authors have indicated the errors in the formulas in the memoirs[2),4),5)].

In the present paper the integral representation of a wave function is reanalyzed and revised.

A function which is treated here is as follows ;

$$P_{-1}(x,y,z) = Re.\left[\frac{1}{2}\int_{-\infty}^{\infty} e^{f(t)}dt\right], \tag{1}$$

where

$$\begin{aligned} f(t) &= ix\cosh t - iy\sinh t\cosh t - z\cosh^2 t, \\ &= ix\cosh t - \rho\cosh t\cosh(t+i\alpha), \end{aligned} \tag{2}$$

and $\rho = \sqrt{y^2+z^2}, \quad \tan\alpha = y/z.$

Then, since we have an integral

$$e^{-\rho\cosh^2(t+i\alpha/2)} = \frac{1}{2\sqrt{\pi\rho}}\int_{-\infty}^{\infty} e^{-\frac{v^2}{4\rho}-iv\cosh(t+i\alpha/2)}dv, \tag{3}$$

inserting this in equation (1) and shifting the path of the integration in the t-plane yields

$$P_{-1}(x,y,z) = \frac{e^{(\rho-z)/2}}{4\sqrt{\pi\rho}}\int_{-\infty}^{\infty} e^{-\frac{v^2}{4\rho}}dv\int_{-\infty}^{\infty} e^{iR\cosh(t+i\alpha/2-i\psi)}dt, \tag{4}$$

where

$$R = \sqrt{x^2+v^2-2xv\cos(\alpha/2)}, \quad \tan\psi = \frac{x\sin(\alpha/2)}{x\cos(\alpha/2)-v}. \tag{5}$$

Making use of the integral representation of Bessel function of the second kind

$$Y_0(R) = -\frac{1}{\pi}\int_{-\infty}^{\infty} e^{iR\cosh u}du, \tag{6}$$

We obtain the following formula :

$$P_{-1}(x,y,z) = -\frac{e^{(\rho-z)/2}}{4\sqrt{\rho/\pi}}\int_{-\infty}^{\infty} e^{-\frac{v^2}{4\rho}} Y_0(R)dv = I_1. \tag{7}$$

This formula is correct in the following special cases, that is,

$$P_{-1}(0,y,z) = \frac{1}{2}e^{-z/2}K_0(\rho/2), \tag{8}$$

$$P_{-1}(x,0,0) = -\frac{\pi}{2}Y_0(x), \tag{9}$$

but does not contain a divergent wave component and is not correct in general. Moreover, equations (8) and (9) are special cases of the following expansions[3].

$$P_{-1}(x,y,z) = \frac{1}{2}e^{-z/2}\sum_{n=0}^{\infty}(-1)^n\epsilon_n K_n(\rho/2)J_{2n}(x)\cos n\alpha, \tag{10}$$

$$P_{-1}(x,y,z) = -\frac{\pi}{2}e^{-z/2}\sum_{n=0}^{\infty}\epsilon_n I_n(\rho/2)Y_{2n}(x)\cos n\alpha, \tag{11}$$

Equation (10) is convergent and gives a correct value in the range of a moderate $x^2/(4\rho)$ value but it is not convergent numerically when the value largely increases.

On the other hand, equation (11) is not convergent but is an asymptotic one. Some authors have discussed its defect which does not give a divergent wave component.

In these circumstances, the present paper aims to revise equation (7).

Now, we evaluate equation (4) using the path of the integration in the t-plane as shown in Fig.1 where the angle ψ takes zero at negative infinity of v and tends to π at positive infinity.

However, the horizontal line $i\pi/2$ in the t-plane is singular for the integrand of the integral, so the pass cannot cross the line $i\pi/2$ which causes the error.

Hence, the absolute value of this angle ψ must be confined within $\pi/2$ in order to correct this point.

Now, if the value of v is complex in equation (4), the argument of the potential term of the integral becomes as follows refering to Fig. 2.

$$R\cosh(t + i\alpha/2 - i\psi) = Re^{i\gamma}\cosh(\tau + t + i\alpha/2 - i\psi), \tag{12}$$

where

$$R = \sqrt{r_1 r_2}, \quad e^{\tau} = \sqrt{r_1/r_2}, \tag{13}$$

$$\frac{\theta_1+\theta_2}{2} = \gamma, \quad \frac{\theta_1-\theta_2}{2} = \psi, \tag{14}$$

Moreover, if we choose the path of the integration as shown in Fig.3 where γ takes zero from negative infinity of v to the point B and takes $-\pi$ from B to positive infinity, the dotted line in Fig.1 becomes the solid line which has a jump at A,B,C.

Thus, we obtain the same form as equation (4), but the argument of Bessel function of the second kind in the integrand must be multiplied by $exp(-i\pi)$.

Since we have the relation [1]

$$Y_0(Re^{-i\pi}) = Y_0(R) - 2iJ_0(R), \tag{15}$$

The real part becomes the same as the integral (7) but we must add the integral on the path A,B,C in Fig.3 where Y_0 cancels out with each other but J_0 remains.

Therefore, equation (4) can be rewritten in the following form.

$$P_{-1}(x,y,z) = I_1 + I_2 \tag{16}$$

where I_1 denotes equation (7) and I_2 is the term to be added. Making use of equation (12) through (15), we obtain I_2 as follows ;

$$I_2 = Re.\left[-\frac{i}{2}\sqrt{\pi/\rho} \times e^{(\rho-z)/2} \int_A^B e^{-\frac{v^2}{4\rho}} J_0(R)dv\right], \tag{17}$$

where $\quad A = x\cos(\alpha/2) \quad , \quad B = xe^{i\alpha/2},$

or

$$I_2 = Re.\left[\frac{1}{2}\sqrt{\pi/\rho}e^{(\rho-z)/2} x \sin\frac{\alpha}{2} \int_0^1 e^{-\frac{v^2}{4\rho}} J_0(R)du\right], \tag{18}$$

where $\quad v = x\cos(\alpha/2) + iux\sin(\alpha/2) \quad , \quad R = x\sqrt{1-u^2}\sin(\alpha/2),$

Now, we examine the result equations (16) and (17) in the following manner. Firstly, when x becomes zero, then I_2 vanishes clearly, and when both y and z tends to zero, then I_2 vanishes owing to the exponential term of the integrand of (14) or (15). Therefore, the formulas (8) and (9) are correct.

Secondly, let us consider an asymptotic character of the integral. We can integrate asymptotically as follows.

$$\int_a^\infty e^{-\frac{v^2}{4\rho}} dv \longrightarrow \frac{2\rho}{a} e^{-\frac{a^2}{4\rho}}, \quad for \ |\frac{a^2}{4\rho}| \gg 1, \tag{19}$$

Then, we can evaluate the integral approximately for small ρ as follows.

$$I_2 \longrightarrow Re.\left[\frac{\sqrt{\pi\rho}}{4x} e^{\frac{\rho-z}{2} - \frac{x^2}{4\rho}e^{i\alpha} + \frac{i(\pi-\alpha)}{2}}\right]. \tag{20}$$

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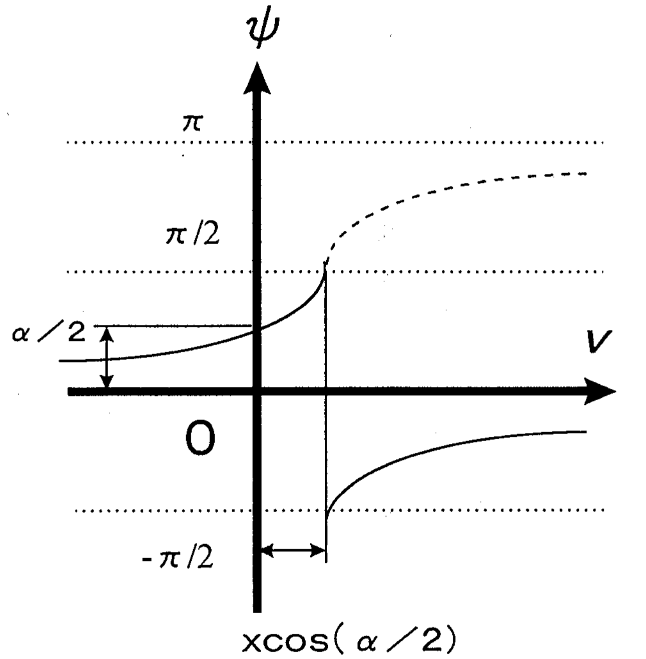

Fig. 1 The value of ψ

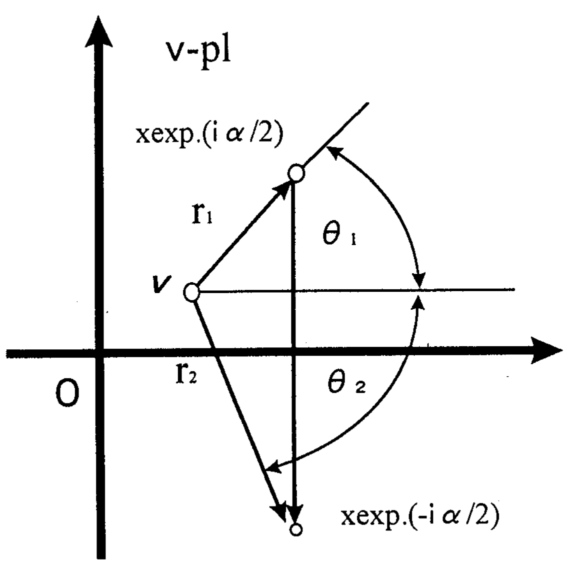

Fig. 2 Complex v-plane

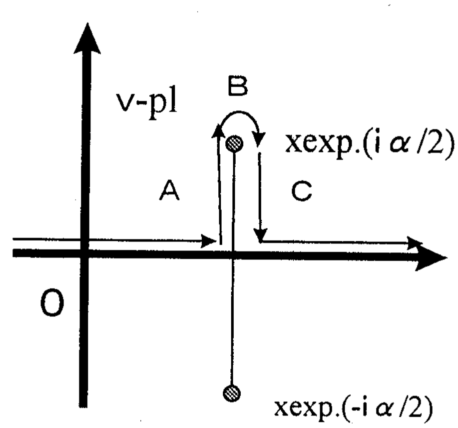

Fig. 3 Path of integration in v- plane

SOME UNEXPLORED ASPECTS OF HYDROFOIL WAVE DRAG

J. P. Breslin

A brief recounting of the milestones in the development of hydrofoil craft from the end of the last century and ending with the vessels built by the US Navy is given. An account of the evolution of theory for invicid flow about hydrofoil sections and the extension to finite aspect ratios is followed by three applications of lifting line theory to a foil tested at in a model basin.

The wave resistance of an aspect ratio 10 hydrofoil as inferred from analysis of lift and drag measurements is compared with results of lifting-line theory for infinitely deep water, for the depth of the towing tank and for a channel of the same width and depth of the test basin. The poorest correlation is obtained for the latter condition. Suggestions for additional work on hydrofoil lift and drag are given.

SLOSHING IN TWO-DIMENSIONAL TANKS

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1. INTRODUCTION

Environmental concern has lead to requirements about double bottoms and skin in new tankers. Since it is desirable to save steel, this has lead to wide oil tanks that can be smooth inside. This increases the danger of occurrence of sloshing and large slamming loads inside the tanks. The most violent fluid motions occur in the vicinity of the lowest natural period for the fluid motion inside the tank. When the tank is smooth, viscous effects are not important and potential flow theory can be used. Nonlinear free surface effects are significant. However, ship motions exciting sloshing are often not large. This means that the external hydrodynamic loads can be approximated by linear theory. However, the coupling between the external linear flow and the internal nonlinear flow should be considered.

There exist commercial CFD codes based on Navier-Stokes Equations and nonlinear free surface conditions that are used to simulate sloshing. A difficulty occurs in describing simultaneously the slamming loads inside the tank. A reason is the much smaller time scale of slamming relative to the characteristic sloshing period. Hydroelasticity may also complicate the simultaneous solution of sloshing and slamming loads.

The complexity of the sloshing flow can easily lead to inaccuracies in the numerical solution. Good verification procedures is therefore of great importance. This paper describes a verification procedure of a nonlinear numerical method for sloshing.

2. THEORY

The method is based on Moiseev's (1958) perturbation method. Details are described by Solaas (1995) and Solaas and Faltinsen (1997). The forced sway or roll motion of the tank is $0(\varepsilon)$ and the fluid response is $0(\varepsilon^{1/3})$. Here ε is a small parameter and a measure of the ratio between the tank motions and the horizontal dimensions of the tank. Non-shallow water depth and two-dimensional flow are assumed. The tank oscillates with frequency ω and a steady-state solution is found. The lowest natural frequency σ_1 for the fluid motion is related to ω by

$$\omega^2 = \sigma_1^2 + \varepsilon^{2/3}\alpha \tag{1}$$

where $\alpha = 0(1)$. The total velocity potential for the fluid motion is expressed as

$$\Phi_T = \phi_1\varepsilon^{1/3} + \phi_2\varepsilon^{2/3} + \phi_3\varepsilon + \phi_c(x,y)\cos\omega t \tag{2}$$

where ϕ_c is of $0(\varepsilon)$, satisfies the body boundary condition, but not the free surface condition. Further t is the time variable. The solution of ϕ_1 can be written as $\psi_1(x,z)\,N\cos\omega t$, where ψ_1 is the eigenfunction for the fluid motion corresponding to the natural frequency σ_1. N is determined by a secularity condition in the 3.order solution. ψ_1 (and ϕ_2 and ϕ_3) are determined by a low order panel method based on representing the velocity potential by Green's second identity. The second order potential ϕ_2 satisfies an inhomogeneous free surface condition which is a function of ϕ_1 and follows by the perturbation scheme. The normal derivative of ϕ_2 on the mean position of tank surface is zero. The solution can be written as

$$\phi_2 = \alpha_0 t + \sum_{n=1}^{\infty} \psi_n(x,z) d^{(n)} \frac{N^2}{2} \sin(2\omega t) \tag{3}$$

Here ψ_n are eigenfunctions for the fluid motion corresponding to eigenfrequency number n. α_0 is a constant and determined by conservation of fluid mass. It follows from this requirement that the perturbation scheme is only possible with vertical walls at the mean waterline. The third order potential ϕ_3 satisfies an inhomogeneous free surface condition which is a function of ϕ_1, ϕ_2 and ϕ_c. The normal derivative of ϕ_3 on the mean position of the tank surface is zero. The right hand side of the free surface includes a term proportional to $\psi_1 \cos\omega t$. This leads to a secularity condition that determines N as the solution of

$$a_1 N^3 + \alpha N + e_1 = 0 \tag{4}$$

This means that up to three solutions of N is possible for any frequency ω.

3. VERIFICATION

Faltinsen (1974) derived an analytical solution for a rectangular tank based on Moiseev's procedure. This was used by Solaas (1995) and Solaas and Faltinsen (1997) to compare all details of the first, second and third order solution. It was found that many elements were needed in the low order panel method. For instance with 500 elements evenly distributed on the mean free surface, the third order potential oscillating with frequency 3ω have relative error of $0(10^{-3})$ on the free surface element closest to the intersection between the near body surface and the mean free surface. This verification of the numerical method demonstrates that great care has to be shown in the numerical analysis.

4. CONCLUDING REMARKS

High numerical accuracy is needed in a numerical method describing sloshing in a tank.

A perturbation solution based on Moiseev's procedure can only be used for tanks with vertical walls at the mean waterline.

A perturbation solution based on Moiseev's procedures seems difficult to generalize to irregular sea.

Sloshing and sloshing induced slamming have very different time scales, which makes an integrated analysis difficult. A possibility may be to generalize the hydroelastic slamming theory described by Faltinsen (1997).

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The CHAPMAN Project

Development of a New Navier-Stokes Solver with a Free Surface

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As shown in the 1990 and 1994 Workshops on hydrodynamics CFD the CFD technique may now be used for many practical purposes in ship design. Several obstacles remain however, as explained in the 18th Weinblum memorial lecture by the present author. The accuracy needs to be improved in resistance prediction and in the computation of the details of the wake field. To accomplish this, improvements may be necessary in the following areas: grid generation, turbulence modelling, free surface boundary conditions and numerics.

CHAPMAN is a cooperative project between Chalmers and FLOWTECH International for developing a new Navier-Stokes solver with improvements in all four areas above. The method uses a structured multi-block overlapping grid generator CHALMESH, developed within the project. Thin curvilinear component grids are employed near the hull and all appendages, and these component grids are embedded into a global Cartesian grid. CHALMESH takes care of the interpolation in the overlapping regions. Singularities are avoided by introducing separate component grids around singularity lines. The propeller is represented in a cylindrical component grid, which rotates inside the hull grid and this will enable the blade flow to be computed when the propeller rotates in the behind condition.

The solver has a free surface capability based on the level set approach, which is capable of handling overturning waves and changes in topology, like when the wave breaks. The Reynolds-averaged Navier-Stokes equations are solved with an advanced turbulence model, and several alternatives for this model are now being tested for some generic test cases. A mixed explicit/implicit temporal solution scheme is under develoment where the implicit technique is used only in the normal direction in the thin curvilinear grids.In this way the small time steps required in the explicit technique due to the very thin cells close to the hull surface are avoided. To minimize numerical dissipation central differencing is used for all terms and the minimum amount of artificial dissipation needed to stabilise the solution for the given grid spacing is computed from a theory for the smallest scales by Henshaw and Kreiss. Alternatively, the theory may be used for finding the required grid spacing for stability without artificial dissipation.

50 YEARS OF YOKOHAMA NATIONAL UNIVERSITY
SHIP HYDRODYNAMICS LABORATORY

BY

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Foreword

In the occasion of the 100th anniversary of the late Professor Georg Weinblum, 50 years of the research activities at Ship Hydrodynamics Laboratory of Yokohama National University, of which I have been in charge since 1947 until my retirement in 1988, is reviewed briefly.

This topic has much relevance to the memory of Prof.Weinblum, because most of the projects carried out in this laboratory have been motivated or stimmulated more or less by his work, especially at their earlier stage. I studied one of his earliest paper published in 1930 on ZAMM[1) in 1944,when I was a student of University of Tokyo, and I was much impressed by his work. This experience had become obviously the motivation of my first work on the study of planing hulls. Another example of his influence is through the work on the hull form of minimum wave resistance which was seemingly the subject of Weinblum's main interest.

The first time when I met Prof.Weinblum was in 1963 in the occsion of International Seminar on Theoretical Wave Resistance at Ann Arbor Michigan. My great surprise at that moment was that he had already known my earlier work. I still remember his encouragement through the work on the wave resistance of slender ships, which I was engaged in at the moment. Since that time, I was able to keep contact with him through the technical committee of ITTC until his death. Therefore the influence from him may appear throughout the period.

The Ship Hydrodynamics Laboratory

Ship Hydrodynamics Laboratory of Yokohama National University belongs to the Department of Naval Architecture and Ocean Engineering, which was founded in 1930 as a part of Yokohama College of Engineering founded in 1920. The College was shifted to Yokohama National University in 1949 by the reformation of the educational system. The university moved to a new campus in1976. Main research facilities of the department is as follows.

Towing Tank:

Old Campus	(1933 - 1976)	L × B × D = 50.4m × 3.6m × 2.75m
New Campus	(1976 - - -)	L × B × D = 110m × 8.0m × 3.5m

Other Facilities:

Circulating Water Channel
Wind Tunnel

On my retirement from Yokohama National University in 1988, Prof. Mitsuhisa Ikehata has succeeded to the position in charge of the laboratory.

Outline of Research Projects

Subjects of the research projects carried out at the Ship Hydrodynamics Laboratory since 1947 are listed below in time sequence. Numbers in parentheses indicate the year when the first paper on the subject in each project was published, and the superscript gives the corresponding literature.

Hydrodynamics of Planing Hulls:

Two Dimensional Problem

Theory of Resistance Components, Spray and Wave (1947)[2)]

Pressure Distribution , Analytical Solution (1951)[3)]

Three Dimensional Problem

Resistance Components (1949)[4)]

High Aspect Ratio Approximation (1953)[5)]

Low Aspect Ratio Approximation (1962)[6)]

Nonlinear Phenomena in Shallow Water:

Aerodynamic Analogy (1952)[7)]

Sinkage and Change of Trim (1981)[8)]

Detection of the Boundary Layer Transition:

Hot Wire Anemometry in the Towing tank (1953)[9)]

Flow Visualization in the Towing Tank (1954)[10)]

Motion of Bodies under Free Surface:

Hydrofoil of Finite Span (1953)[11)]

Non-uniform Motion of a Submerged Body (1955)[12)]

Wave Force on an Obstacle:

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Concluding Remarks

Half a century has passed since the Hydrodynamic Laboratory of Yokohama National University started. One may observe its research activities to have covered various field of ship hydrodynamics. On arranging research projects, it has been intended to keep an even share between theory and experiment. This criterion seems to have been nearly attained. Another idea is that the practical usefulness, especially in the field of shipbuilding, of theoretical findings has been taken seriously. This concept may match the spirit of Prof. Weinblum.

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"There is no theorem like the Lagally theorem." "The Ellipsoid is God's gift to naval architects."

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Georg P. Weinblum (1897-1974) was born about the same time when John H. Michell (1863-1940) had already completed writing his seminal work on the wave resistance of thin ships (Michell 1898) - some hundred years ago. Thus, it was not a coincidence that G.P.W. chose to write his doctoral dissertation (submitted in 1929) on ships of minimum wave resistance using Michell's theory. It is also well known that Michell's famous article did not get the proper attention (at least from the naval architects community) for over 25 years, although it was published in one of the most prestigious journals (Philosophical Magazine). Indeed it was Sir Thomas Havelock, probably the leading theoretician working on ship hydrodynamics at the beginning of the century, who rediscovered Michell's paper 25 years after it had appeared (Havelock 1923). The first reference to Michell's work in Havelock's paper appeared only as a side comment "On the other hand, Michell, in an extremely interesting paper, gave a general expression for wave resistance, but it suffers from serious limitations, in that the surface of the ship must be everywhere inclined at only small angle to its vertical meridian plane". A more well deserved credit to Michell's theory was given in a paper by Wigley (1926). It is believed that by that time Weinblum became acquainted with Michell's work and since then he became a strong advocate and promoter for using the Michell's wave resistance formula. Weinblum also tried to bridge the gap between theoreticians and naval architects practitioners and provided in his papers sample computations and comparisons between theory and experiments. As an example, we mention his joint paper with Graff & Kracht on the wave resistance of a conventional merchant-ship hull which includes a comparison of drag measurments with numerical evaluations of the Michell integral (Graff et al. 1964). In his continuous efforts to exploit relevant theories to find how they can help ship designers, he has introduced, during the four year period (1948-1952) that he spent at the DTMB, the important paper of Lagally (1923) to the U.S. community of ship hydrodynamicists. In this context we cite a paragraph from Landweber's paper (1967) who wrote "About 20 years ago Georg Weinblum succeeded in convincing his incredulous colleagues at the David Taylor Model Basin that THERE IS NO THEOREM LIKE THE LAGALLY THEOREM ... and pointed out the power of the LAGALLY theorem and new fields of research to many of us". Yet another off-repeated statement of Weinblum is "THE ELLIPSOID IS GOD'S GIFT TO NAVAL ARCHITECTS" (Newman 1972). Weinblum suggested to use the concept of equivalent ellipsoids for approximating real ship forms (Weinblum 1936). He was definitely inspired by the theoretical work of Havelock and was probably the first worker in ship theory to study and apply the hydrodynamics of spheroids and ellipsoids to more general bodies of revolution. The same citation regarding ellipsoidal forms, which is attributed to Weinblum, is also mentioned in Wu & Chwang (1974) and Miloh (1979). Stimulated by these two Weinblum quotations, we intend to present here an historical account of some theoretical methods for calculating potential flows about 3-D ellipsoidal shapes. Also presented is the development of the Lagally method for calculating hydrodynamic loads on 3-D arbitrary rigid and defformable moving bodies.

Ellipsoid Theorem

In order to determine the hydrodynamical loads on a moving body by using the Lagally theorem, it is necessary first to find the image singularity system within the body of the exterior potential flow field. For a general body this procedure usually involves solving numerically an integral equation of a Fredholm type. However, for the class of symmetric separable quadratic surfaces (i.e. spheres, spheroids and ellipsoids), the image singularity system can be found analytically using harmonic analysis. The idea is to analytically continue the exterior flow across the surface inside the body and to find the interior ultimate (minimal) singularity system.
Let us first consider a spherical coordinate system (R, μ, ψ) defined by

$$x = R\mu, \quad y + iz = R(1-\mu^2)^{\frac{1}{2}} e^{i\psi}, \tag{1}$$

where (x, y, z) is a cartesian system. An arbitrary exterior potential flow field about a sphere which vanishes at infinity $R \to \infty$, can be represented by a Neumann series of exterior spherical harmonics. Following Hobson (1955) a typical term of such a series can be written as

$$R^{-(m+1)} P_n^m(\mu) e^{i\psi} = \frac{(-1)^n}{(n-m)!} \left(\frac{\partial}{\partial x}\right)^{n-m} \left(\frac{\partial}{\partial y} + i\frac{\partial}{\partial z}\right) (x^2+y^2+z^2)^{-\frac{1}{2}}, \tag{2}$$

where $P_n^m(\mu)$ denotes the Legendre polynomial. Thus, the ultimate system of singularities for exterior spherical harmonics consists of a system of multipoles at the origin. A similar theorem for spheroidal exterior harmonics has been given without proof by Havelock (1952). A proof was provided later on by Miloh (1974). The orthogonal transformation between a cartesian and a spheroidal coordinate system (ξ, μ, ψ) is

$$x = \xi\mu, \quad y + iz = (\xi^2 - 1)^{\frac{1}{2}}(1 - \mu^2)^{\frac{1}{2}} e^{i\psi}, \tag{3}$$

where the distance between the two foci is taken to be 2. Havelock theorem can then be written as

$$Q_n^m(\xi) P_n^m(\mu) e^{im\phi} = \frac{1}{2}\left(\frac{\partial}{\partial y} + i\frac{\partial}{\partial z}\right)^m \int_{-1}^{1} \frac{(\xi^2 - 1)^{\frac{m}{2}} P_n^m(\xi)\ d\xi}{\sqrt{(x - \xi)^2 + y^2 + z^2}}, \tag{4}$$

where Q_n^m represents the Legendre polynomial of the second kind. Thus, the ultimate image singularity system for an exterior spheroidal harmonic can be represented as a multipole distribution along the axis between the two foci. The axisymmetric case which corresponds to $m = 0$, renders a source distribution $P_n(x)$ on $1 > |x|$. The most general separable quadratic surface is the triaxial ellipsoid.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1; \quad a > b > c. \tag{5}$$

The orthogonal transformation between the cartesian (x, y, z) and the ellipsoidal coordinate system (ρ, μ, ν) is given by

$$x = \frac{\rho\mu\nu}{hk}, \quad y^2 = \frac{(\rho^2 - h^2)(\mu^2 - h^2)(h^2 - \nu^2)}{h^2(k^2 - h^2)}, \quad z^2 = \frac{(\rho^2 - k^2)(k^2 - \mu^2)(k^2 - \nu^2)}{k^2(k^2 - h^2)}, \tag{6}$$

where $h^2 = a^2 - b^2$, $k^2 = a^2 - c^2$ and $-h < \nu < h < \mu < k < \rho < \infty$. An arbitrary potential flow field past an ellipsoid can be represented in terms of ellipsoidal exterior harmonics $F_n^m(\rho)E_n^m(\mu)E_n^m(\nu)$ where E_n^m and F_n^m denote the Lame polynomials of the first and second kind respectively. There exist four different types of Lame polynomials of the first kind; class K and L (both even in z) and class M and N (both odd in z). The ellipsoidal theorem (Miloh 1974) then states that exterior ellipsoidal harmonics of class K and L may be generated by a source distribution $\sigma(x, y)$ such that

$$F_n^m(\rho)E_n^m(\mu)E_n^m(\nu) = -\int_{S_0} \frac{\sigma(x', y')\ dx'\ dy'}{\sqrt{(x - x')^2 + (y - y')^2 + z^2}}, \tag{7}$$

where (x', y') are the rectangular points in the (x, y) plane within the focal ellipse (the ultimate image system)

$$S_0 \equiv \frac{x^2}{a^2 - c^2} + \frac{y^2}{b^2 - c^2} = 1, \quad z = 0. \tag{8}$$

and $\sigma(x', y')$ is given by

$$\sigma(x', y') = -\frac{(2n+1)E_n^m(\mu')E_n^m(\nu')}{2\pi k E_n^m(k)\sqrt{(k^2 - h^2)}}\left(1 - \frac{x^2}{k^2} - \frac{y^2}{k^2 - h^2}\right)^{-\frac{1}{2}}. \tag{9}$$

In a similar manner, one can express an exterior ellipsoidal harmonic of class M or N as a normal doublet distribution

$$F_n^m(\rho)E_n^m(\mu)E_n^m(\nu) = \frac{\partial}{\partial z}\int_{S_0} \frac{\delta(x', y')\ dx'\ dy'}{\sqrt{(x - x')^2 + (y - y')^2 + z^2}}, \tag{10}$$

where

$$\delta(x', y') = \frac{(2n+1)E_n^m(\mu')E_n^m(\nu')}{2\pi k \bar{E}_n^m(k)\sqrt{(k^2 - h^2)}}, \quad \bar{E}_n^m(k) = \frac{E_n^m(k)}{\sqrt{\rho^2 - k^2}}. \tag{11}$$

Finally, solving general potential flow problems past ellipsoidal bodies requires the expansion of the Green function $\frac{1}{r_{PQ}}$ (i.e. the inverse of the distance between two points $P(\rho, \mu, \nu)$ and $Q(\rho', \mu', \nu')$ where $\rho' > \rho$) in terms of ellipsoidal harmonics. Such an expansion has been given in Miloh (1973)

$$\frac{1}{r_{PQ}} = \sum_{n=0}^{\infty}\sum_{m=1}^{s(n)} \frac{2\pi}{(2n+1)\gamma_n^m} F_n^m(\rho')E_n^m(\mu')E_n^m(\nu')E_n^m(\rho)E_n^m(\mu)E_n^m(\nu), \tag{12}$$

where

$$\gamma_n^m = \int_{-h}^{h}\int_{h}^{k} \frac{(\mu^2 - \nu^2)(E_n^m(\mu)E_n^m(\nu))^2\ d\mu\ d\nu}{\sqrt{(\mu^2 - h^2)(\mu^2 - k^2)(\nu^2 - h^2)(\nu^2 - k^2)}} \tag{13}$$

and $s(n)$ is defined below for the four classes of Lame polynomials;

$$s(n) = \begin{array}{cccccc} & K & L & M & N & * \\ & 1 + \frac{n}{2} & \frac{n}{2} & \frac{n}{2} & \frac{n}{2} & \text{for } n \text{ even} \\ & \frac{1}{2}(n+1) & \frac{1}{2}(n+1) & \frac{1}{2}(n+1) & \frac{1}{2}(n-1) & \text{for } n \text{ odd} \end{array} \tag{14}$$

Lagally Theorem

Once the image singularity system of the exterior flow field past the body is known, one can directly compute the hydrodynamical loads experienced by the body in terms of these singularities. The so-called Lagally theorem is valid for both $2-D$ and $3-D$ deformable or rigid surfaces and for a line, surface, volume or discrete singularity distribution. Using this technique avoides the computation of the pressure distribution and its integration over the body surface. In many respects it is more direct and accurate than the method of pressure integration and may be also considered as an extension of the $2-D$ Blasius method for $3-D$ flows. Lagally (1922) gave only an expression for the force acting on a source of output m and on a doublet $\mathbf{d}$ both placed in a potential steady stream $\mathbf{v}$. The corresponding Lagally force is $-4\pi\rho[m\mathbf{v} + (\mathbf{d}\cdot\nabla)\mathbf{v}]$ where ρ is the density of the fluid. It is also interesting to note that the particular expression for a point source has been derived earlier by Munk (1921). The so-called steady Lagally method has been revised by Betz (1932) who also provided a simpler derivation. Extensions for unsteady flows and multipoles have been first proposed by Cummins (1953, 1957). Further work on the subject of rigid body hydrodynamics is due to Landweber & Yih (1956) and Landweber (1967). The case of deformable bodies and the appropriate generalization of the Lagally theorem have been discussed and presented by Landweber & Miloh (1980). More recent applications for the case of a moving deformable body embedded in a non-uniform ambient flow field are given in Galper & Miloh (1994, 1995).
Let the equation of the deformable surface in a body-fixed coordinate system be given by $S(\mathbf{r},t)$. Then the deformable potential is found from the following Neumann boundary condition on S

$$\frac{\partial\phi_d}{\partial n} = -\frac{\partial S}{\partial t}\frac{1}{|\nabla S|}. \tag{15}$$

Assume next that the image singularity system consists of multipoles m_q of order $q = \alpha+\beta+\gamma$ located at (x_s, y_s, z_s) where the internal flow field is given by

$$\phi = -m_q D_q(\frac{1}{R}), \quad D_q = \frac{\partial^q}{\partial x_s^\alpha \partial y_s^\beta \partial z_s^\gamma} \quad, R^2 = (x-x_s)^2 + (y-y_s)^2 + (z-z_s)^2. \tag{16}$$

The Lagally force acting on the deformable body is then given by

$$\frac{\mathbf{F}}{\rho} = \frac{d}{dt}\left(v(t)\mathbf{V}_c - 4\pi\sum_s^{(d)} m_q D_q(\mathbf{r})_s + \mathbf{K}_d\right) - 4\pi\sum_s m_q D_q(\nabla\phi)_s, \tag{17}$$

where $v(t)$ is the volume of the body, $\mathbf{V}_c$ is the instantaneous velocity of its centroid and $\mathbf{K}_d$ is the deformation Kelvin impulse defined by

$$\mathbf{K}_d = -\int_S \phi_d\, \mathbf{n}\, dS. \tag{18}$$

Also $\sum_s$ denotes the sum of all singularities and $\sum_s^{(d)}$ excludes those due to ϕ_d. The above formulation can be applied to the problem of self-propulsion of a deformable body which was first discussed in Benjamin & Ellis (1966, 1990), Saffman (1967), Wu (1976) and Miloh (1983). It has been demonstrated in these papers that a deformable body can propel itself persistently starting from rest in an inviscid and incompressible fluid by applying a periodic surface deformation with zero-mean. The collinear velocity of self-propulsion U_s can be expressed in terms of the deformation Kelvin-impulse K_d, the body mass M_b and its added-mass T as

$$(M_b + T)U_s + K_d = 0. \tag{19}$$

It is shown that the persistent self-propulsion motion arises from a non-linear interaction between symmetric and skew-symmetric surface deformation modes. Extension for the case of a maneuvering body (i.e. including auto-rotation) and self-propulsion in an ambient non-uniform stream, are given in Miloh & Galper (1993) and Galper & Miloh (1995). It is demonstrated that the presence of a flow non-uniformity may considerably amplify the order of magnitude of the self-propulsion velocity, as a result of parametric resonant interactions between surface deformations and flow non-uniformity. Applications to bubble dynamics including the problem of bubble coalesce in a cloud are discussed within the same framework in Galper & Miloh (1994, 1995). The same methodology can be also used for estimating the hydrodynamical loads on slender ocean structures in a non-uniform wave field (Galper & Miloh (1996, 1997)).

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Resonant Diffraction Problems

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Resonant motions of floating bodies are particularly important when the damping is small. Familiar examples include rolling of ships, and the heave response of slender spar buoys. In cases such as these the resonant motion is associated solely with the body dynamics, that is to say with the force or moment coefficients of the radiation problem where the body is oscillating in otherwise calm water. The resonant frequency is determined from the condition that the inertial force due to the body mass and hydrodynamic added mass is equal and opposite to the hydrostatic restoring force; the amplitude at resonance is inversely proportional to the damping.

Resonant motions of the free surface can occur independently of the body motions for certain special types of diffraction problems. Well known examples include moon pools, and wave-power devices with oscillating water columns, where an enclosed internal region of the fluid exists with a free surface, coupled to the exterior domain via a submerged opening. The lowest resonant frequency is associated with the Helmholtz mode, where the motion of the internal fluid is similar to a heaving rigid body with the same mass and waterplane area.

The case of a moon pool is particularly important for certain types of offshore platforms. Computations are presented to illustrate the amplitude of free-surface elevation at the center of the moon pool, for a generic family of axisymmetric cylinders. The Helmholtz resonance is a prominent feature in the diffraction solution, with increasing peak amplitude and decreasing bandwidth as the moon pool radius is reduced. In the case where the body is free to heave in the presence of incident waves, we find from careful computations that there is no amplification of the moon pool response at the original resonant frequency of the diffraction problem, apparently because the free motions of the body adapt to and cancel out any large forcing pressure at the bottom of the moon pool. Instead, the moon pool resonance occurs at a slightly higher frequency and wavenumber. This is due to the occurrence of a second heave resonant frequency, which in turn is caused by the rapid variation of the heave added mass with respect to frequency.

Complete enclosure of the internal free surface is not necessary. Resonant motions, including the Helmholtz mode, can occur when there is an opening between the interior and exterior fluid regions, as in the case of a harbor with a small entrance (Mei, 1977). Another interesting example is where two vessels are close together in a catamaran configuration or, equivalently, a single vessel is close to a parallel wall (quay). In the long narrow interior domain resonant standing waves can occur with large amplitude, provided the frequency is such that the nodes of the standing wave coincide with the openings to the exterior domain at the ends of the two vessels. Numerical results to illustrate this phenomenon were presented by Newman and Sclavounos (1988).

At the last Workshop Maniar & Newman (1996) showed that resonant motions can occur in the gaps between adjacent circular cylinders in a long periodic array, although there is no clear distinction between the interior and exterior domains of the free surface. More extensive results and analysis are described by Maniar & Newman (1997). These resonant modes are associated with trapped waves which exist for diffraction past a single cylinder in a channel, but the connection with that problem is essentially mathematical and cannot be explained on a simple physical basis. This phenomenon is important even for small numbers of cylinders, as in the case of a tension-leg platform, but it is remarkably large for longer arrays with peak wave loads acting on individual cylinders which are more-or-less proportional to the total number of cylinders in the array.

Both 'Neumann' and 'Dirichlet' trapped modes exist in correspondence with the boundary conditions imposed on the walls in the channel problem. The results for long arrays of cylinders also display secondary peaks and intermediate minima, just below these critical frequencies. In recent work Maniar has shown that the secondary features can be explained in terms of superposing end-to-end the diffraction fields of smaller arrays with one-half, one-third, one-quarter, etc. of the total number of cylinders.

Porter and Evans (1997) have shown that analogous resonant modes can occur in the case of a circular array, especially when the gaps between adjacent cylinders are small. At first glance one might suppose that this phenomenon is more analogous to the case of a harbor with a small entrance, where the resonance is associated with the interior fluid domain and free surface. However the correspondence of their modes and wavenumbers with those found for the linear array suggests that the resonance is due to the gaps between the cylinders and not to the interior domain. Indeed, their findings help to explain why this phenomenon is relevant to tension-leg platforms.

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ON SHIP WAVES AT TRANSCRITICAL SPEEDS

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Summary

In a historical review extending back to the memorable International Symposium on Ship Theory held at Hamburg in 1962 to celebrate Georg Weinblum's 65th birthday it was shown how efficiently modern shallow-water ship-wave theory, developed largely by several Weinblum Memorial Lecturers attending this present meeting, has succeeded in explaining various exciting transcritical flow phenomena, originaly observed at full scale more than a century ago and repeatedly verified in ship model tanks. These include the dramatic rise and fall of wave resistance, reversal of squat, metamorphosis of wave pattern, and generation of forward solitons, all occurring as ship speed rises through its critical value in shallow water, particularly in a narrow channel. Systematic model experiments initiated by Weinblum at the Shallow Water Towing Tank in Duisburg (VBD) more than 35 years ago have proved invaluable for validating recent theoretical computations. New wave pattern, side force and yaw moment measurements have corroborated the calculations in more detail. Further development of the theory by this Speaker's group at Duisburg has culminated in the discovery of "superconductive" channels and catamarans, characterized by zero wave resistance at a chosen supercritical design speed. This is achieved, in principle, by mutual cancelation of bow and stern waves, a bit akin to the classical Busemann's biplane proposed for hypersonic flight some 60 years ago. The superconductive catamaran, rendered independent of channel sidewalls by use of suitably cambered hulls, would, besides saving propulsive power by virtue of its vanishing wave resistance, have the additional environmental benefit of being a "no-wash" vehicle. Ongoing research is concerned with the conception of a cambered air-cushion catamaran, ideally eliminating the local wave also.

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MULTIHULLS

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A discussion of various problems involving one or more bodies at or near a free surface is given. The general problem of multihull wave resistance is discussed, including the generalised Michell integral and Krein's zero-drag caravans. Some work done at Adelaide over the past two years on minimising the total (viscous plus wave) drag of multihull ships using the genetic algorithm technique is summarised. Recent work on a pair of tandem submerged cylinders is also discussed, including identification of configurations having zero drag on each separate body.

Remarks on Energy Transport in Waves

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__Preface__ *In 1950 I arrived at the Taylor Model Basin in Washington, age 24, to begin work in naval hydrodynamics. George Weinblum was 53 then. He had arrived there 2 years earlier after a stay in England, and was to return to Hamburg two years later. He was a large man, with a very large head and twinkly eyes, of immense charm and diplomacy and talent, of great sharp wit, and with an international view of life. He seemed somewhat the bohemian.*

He had already made a large impact on the very talented people there, including John Wehausen, Manley St. Denis, Lou Landweber (our Boss), Phil Eisenberg, Bill Cummins, John Breslin, Dick Couch, and others. He loved young people, and he went out of his way to encourage us. His deep faith in the necessity to treat naval architecture problems in a scientific way made a deep and lasting impression on us all, especially considering that he was a man of practical experience.

Despite the tentativeness of life away from his family and homeland, and without a fixed position, I believe that during this period his life was a very happy one. He was well liked by everyone and loved by more than a few persons.

In tribute to him I want to point out that he had a very considerable positive influence on people there, who themselves went on to have a great effect on naval architecture and naval hydrodynamics in our country, and on education in those fields.

Remarks

Is it possible to say anything new about this subject, which is in all relevant texts covered by introduction of the notion of group velocity? Now it is true that the subject of the group velocity includes puzzling aspects. For example, the connection between the parallel and separate treatments of group velocity via kinematical and/or dynamical demonstrations, leading in the case of linear waves to identical results.

And, beyond that, the question as to the proper treatment and results in the case of finite amplitude waves.

The present remarks are however not concerned directly with these questions. Our concern is even more basic. It can be put in this question: What is it that physically propagates at the group velocity? Obviously the difficulty in answering lies in the fact that the energy in surface waves is compartmented in two parts: kinetic and potential. It has been customary to treat these as a sum, and it is the sum of these which demonstrations suggest are propagated at the group velocity. Furthermore, the kinetic energy has been treated not only as averaged in time, but also in the vertical direction.

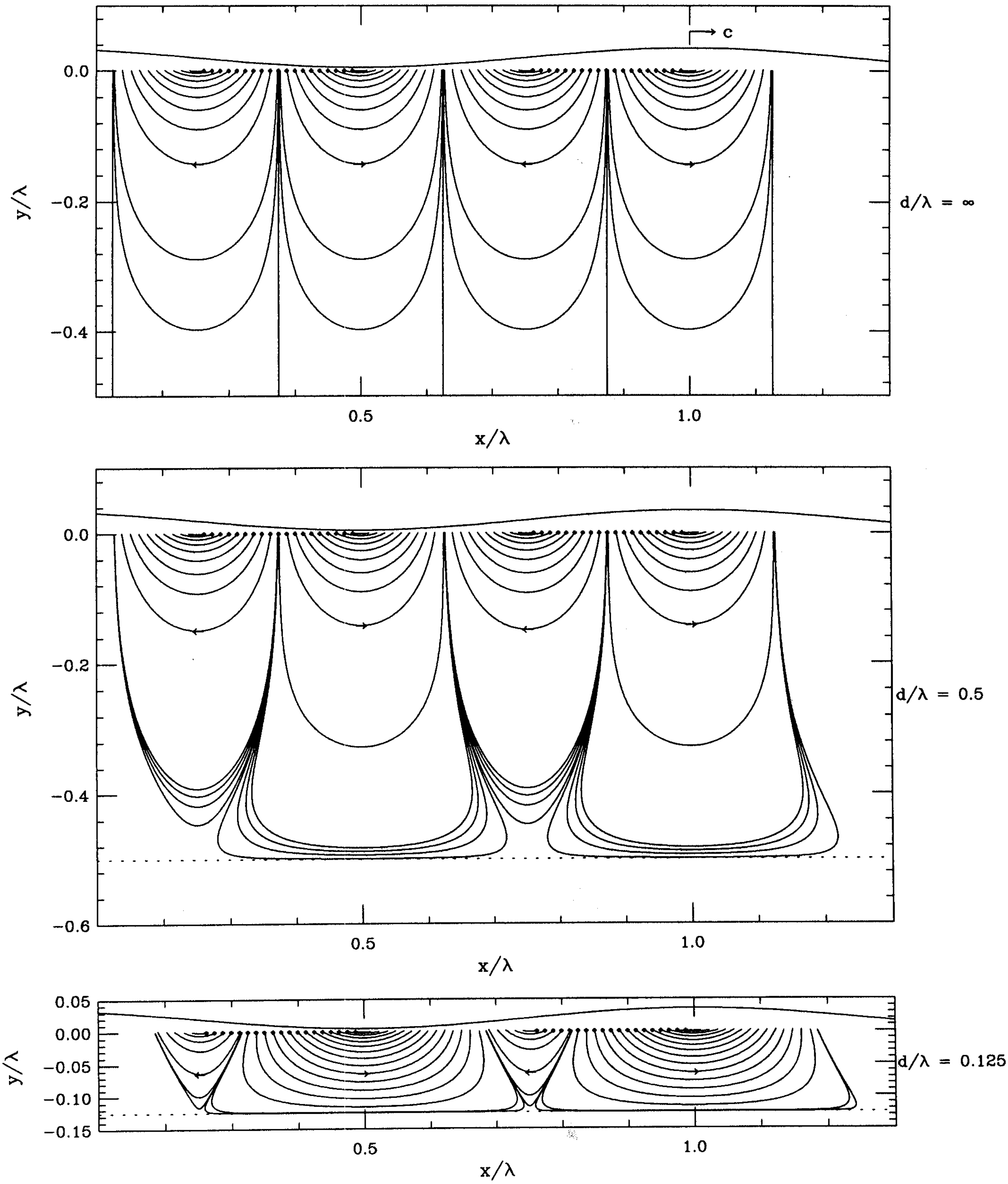
Unfortunately, this customary treatment hides from our view the real mechanisms of energy transport in water waves, and obscures the real meaning of group velocity, which in actuality is an arithmetic mean. The answer to the question underlined above is: only the modulated wave envelope propagates physically at the group velocity.

Proper understanding of the subject requires consideration of the kinetic energy flux vector at all points in the wave, and separate consideration of the surface energy, which itself consists of two parts, gravitational and surface tension. It is also necessary to conceptualize the waves not as a uniform Stokes wave, but as a wave whose amplitude is changing in space and time. It is only when these things are done that the actual mechanisms of wave energy transport reveal themselves clearly.

Then it can be shown that in the case of monochromatic gravity waves, the time averaged kinetic energy at every depth below the wave trough propagates horizontally at speeds between one and two times the phase speed of the wave, depending on the water depth. In a frame moving with the wave speed, the kinetic energy at each point can be seen to propagate along flux lines which extend beneath the surface from one point to another on the wave surface. these lines are arranged in four cells per wavelength, Figure 1, and in shallow water the cells containing flux in the wave propagation direction are more dominant, resulting in a net forward flux relative to the moving wave.

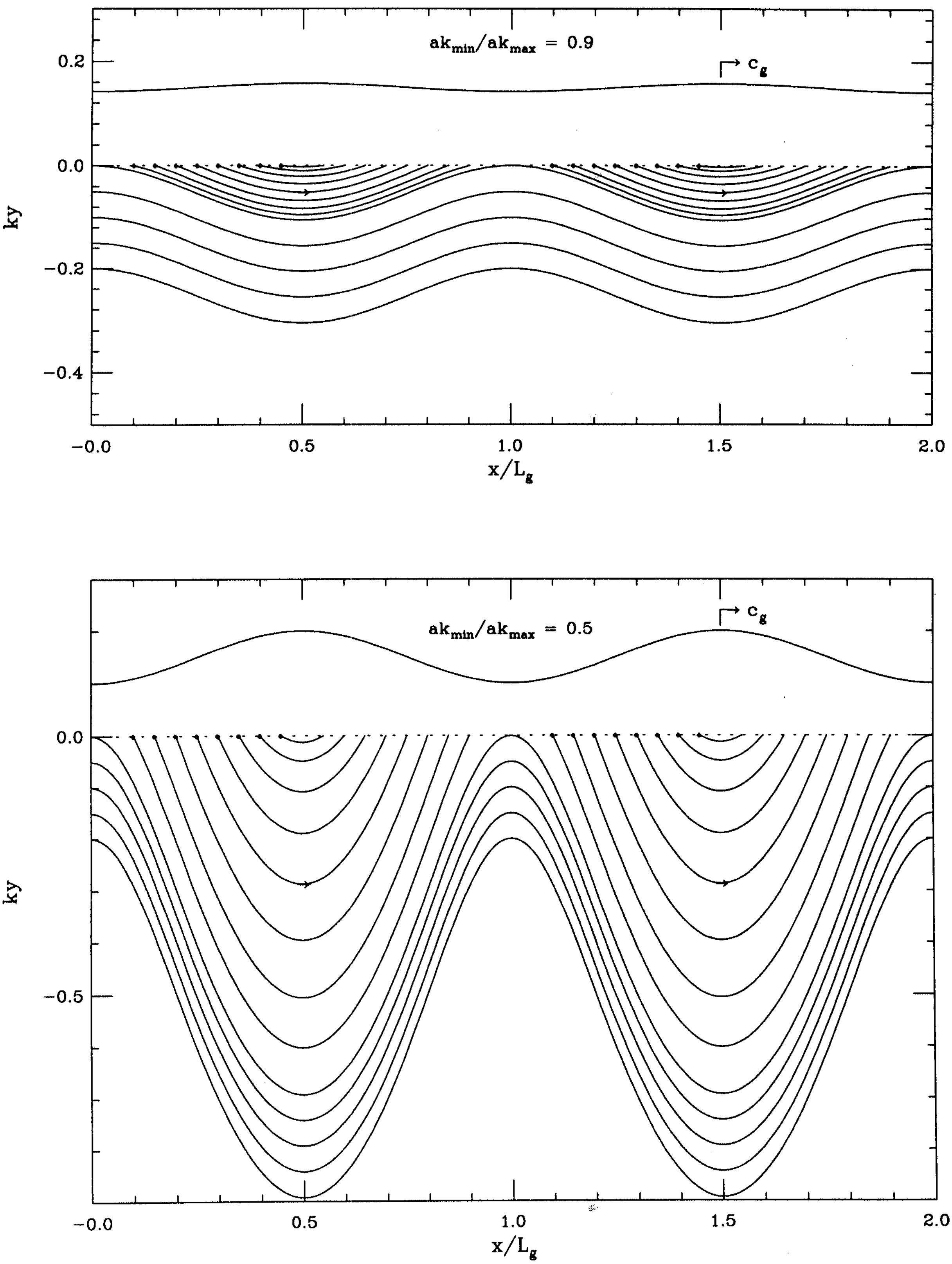
For all waves, the time averaged gravitational potential energy does not propagate at all, while the (linearized) surface tension energy propagates at twice the wave phase velocity. The net result is that the transport speed calculated as a weighted average of the transport of the separate components of energy yields the familiar group velocity.

In the case of a modulating gravity wave the kinetic energy propagates vertically as well as horizontally and in this way provides the potential energy at the surface to the forward face of a wave group and extracts it from the surface in the opposite case, Figure 2. It is due to this mechanism of exchange between gravitational and kinetic energies that the modulation is allowed to propagate, and the speed of propagation for weak waves is precisely the group velocity.



Kinetic Energy Flux under Monochromatic Waves

Figure 1



Kinetic Energy Flux in Wave Groups

Figure 2

NOTES ON WAVE MOTION NEAR A SPHERE BETWEEN PARALLEL WALLS

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1 Introduction

The following problem was proposed to me by G.X. Wu at the 1996 Workshop in Hamburg . A submerged sphere of radius a is placed with its centre at depth f midway between parallel vertical walls $x = \pm\ell$, where $\ell > a$, and performs prescribed simple harmonic oscillations of angular frequency ω and small amplitude. How can the motion be calculated ? Problems of this type in two dimensions are well understood , how can the methods be generalized ? It will be seen that the three-dimensional solution involves much mathematical analysis, and only an outline will be given here.

Rectangular cartesian axes (x, y, z) are taken with the origin O in the mean free surface $y = 0$, where y increases with depth. Let the corresponding velocity potential be denoted by $\phi(x, y, z)\exp(-i\omega t)$. (The time-factor $\exp(-i\omega t)$ will henceforth be omitted.) Then the governing equation is

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\phi(x, y, z) = 0, \tag{1.1}$$

with the boundary condition

$$K\phi + \frac{\partial \phi}{\partial y} = 0 \text{ on } y = 0, \tag{1.2}$$

where $K = \omega^2/g$, and the boundary condition

$$\frac{\partial \phi}{\partial x} = 0 \text{ on } x = \pm\ell. \tag{1.3}$$

We take spherical polar coordinates (r, θ, α) about the centre $(0, f, 0)$ of the sphere, such that

$$x = r\sin\theta\sin\alpha, y = f + r\cos\theta, z = r\sin\theta\cos\alpha, \quad \text{where } r^2 = x^2 + (y-f)^2 + z^2. \tag{1.4}$$

The boundary condition on the sphere $r = a$ is assumed to be of the form

$$\frac{\partial \phi}{\partial r} = U(\theta, \alpha) = \sum_{n=0}^{\infty}\sum_{m=0}^{n} U_n^m \mathrm{P}_n^m(\cos\theta)\cos m\alpha, \text{ say }, \tag{1.5}$$

where $U(\theta, \alpha)$ is a prescribed even function of α, and the coefficients U_n^m are therefore assumed known. (Odd functions of α can be treated in the same way.) The functions P_n^m are the usual associated Legendre functions, see [Bateman, I, 1953]. There is also a radiation condition at infinity: the waves travel outwards towards $z = \pm\infty$.

2 Outline of the solution

The solution will make use of the method of multipoles. A typical solution of Laplace's equation singular at the centre of the sphere is $r^{-n-1}\mathrm{P}_n^m(\cos\theta)\cos m\alpha$. In the absence of side walls the multipole potential (including the image in the free surface) can be shown to be

$$\begin{aligned}(G_n^m)_\infty &= G_n^m(x, y, z; 0, f, 0; \infty)\\ &= \frac{\mathrm{P}_n^m(\cos\theta)}{r^{n+1}}\cos m\alpha + \frac{(-1)^n}{(n-m)!}\int_0^\infty \frac{k+K}{k-K}\, k^n\, e^{-k(y+f)} J_m(k\rho)\, dk \cos m\alpha,\end{aligned} \tag{2.1}$$

where $J_m(Z)$ is the usual Bessel function of order m, see [Bateman, II, 1953], and where the radiation condition is satisfied if the contour of integration passes below the pole $k = K$. When side walls are present then each multipole (2.1) has an image potential $(G_n^m)_{image}$ in the side walls. We write

$$(G_n^m)_\ell = (G_n^m)_\infty + (G_n^m)_{image},$$

Evidently near the centre of the sphere the image potential $(G_n^m)_{image}$ must have an expansion of the form

$$\sum_{N=0}^{\infty} \sum_{M=0}^{N} a(n, m; N, M)\, r^N\, \mathrm{P}_N^M(\cos\theta)\, \cos M\alpha. \tag{2.2}$$

For the solution of our boundary-value problem the coefficients $a(n, m; N, M)$ in (2.2) must be known explicitly, and our recent work has shown that the coefficients in this expansion can actually be found in a form involving single and double integrals. We shall now assume (and it should not be difficult to prove) that the solution of our problem can be expressed as the sum

$$\phi(x, y, z) = \sum_{n=0}^{\infty} \sum_{m=0}^{n} C(n, m) a^{n+1} (G_n^m)_\ell, \tag{2.3}$$

where the coefficients $C(n, m)$ are to be determined from the boundary condition on the sphere. This will be satisfied if the coefficients $C(n, m)$ satisfy a doubly infinite system of the form

$$C(n, m) + \sum_{N=0}^{\infty} \sum_{M=0}^{N} \left(\frac{a}{\ell}\right)^{N+n+1} b(n, m : N, M)\, C(N, M) = U_n^m, \tag{2.4}$$

and in this system the coefficients $b(n, m; N, M)$ are known explicitly as double or single integrals. These coefficients depend on n, m, N and M, and also on the parameters Kf and f/ℓ, and it remains to compute them numerically. The doubly-infinite system (2.4) must then be solved for each set of values of the three parameters Kf, f/ℓ and a/ℓ. It should not be difficult to show that this system has a solution in principle, except possibly at a certain discrete set of frequencies corresponding to trapped modes.

3 Construction of the multipole potentials $(G_n^m)_\ell$

In the construction of the multipoles $(G_n^m)_\ell$ an important part is played by various forms of the Havelock wavemaker theory, see [Havelock, 1929]. Havelock expansions are expansions in which the variation in the y-direction is expressed in the form

$$f(y) = Ae^{-Ky} + \int_0^\infty B(k)(k \cos ky - K \sin ky)\, dk, \tag{3.1}$$

where

$$A = 2K \int_0^\infty f(y) e^{-Ky}\, dy, \tag{3.2}$$

and

$$B(k) = \frac{2}{\pi(K^2 + k^2)} \int_0^\infty f(y)(k \cos ky - K \sin ky)\, dy. \tag{3.3}$$

This expansion is associated with the boundary condition (1.2) . Here we shall consider only the case $n = m = 0$ which is typical, and we shall write G_∞ and G_ℓ in place of $(G_0^0)_\infty$ and $(G_0^0)_\ell$. As has already been seen in (2.1), we find that

$$G_\infty = \frac{1}{r} + \int_0^\infty e^{-k(y+f)} \frac{k + K}{k - K} J_0(k\rho) dk, \tag{3.4}$$

where it can be shown that the radiation condition is satisfied if the path of integration passes below the pole $k = K$. We also need two Havelock expansions for G_∞ in rectangular coordinates. One of these is valid for $x > 0$, the other is valid for $x < 0$. These can be found explicitly, either by deforming the contour of integration in (3.4) or by using Green's Theorem. It is found that

$$G_\infty(x,y,z) = 2Ke^{-K(y+f)} \int_{-\infty}^{\infty} \frac{d\nu}{(\nu^2-K^2)^{1/2}} \exp\{-|x|(\nu^2-K^2)^{1/2}\} e^{-i\nu z} \tag{3.5}$$

$$+ \frac{2}{\pi}\int_0^\infty \frac{dk}{K^2+k^2}(k\cos ky - K\sin ky)(k\cos kf - K\sin kf) \times$$

$$\times \int_{-\infty}^{\infty} \frac{d\nu}{(\nu^2+k^2)^{1/2}} \exp\{-|x|(\nu^2+k^2)^{1/2}\} e^{-i\nu z}, \tag{3.6}$$

where the contour of integration in (3.5) passes above $k = -K$ and below $k = K$. We next suppose that side walls are present. Then the image potential must have a Havelock expansion of the form

$$G_{image}(x,y,z) = e^{-Ky} \int_{-\infty}^{\infty} d\nu\, A(\nu)\, \cosh\{x(\nu^2-K^2)^{1/2}\} e^{-i\nu z} \tag{3.7}$$

$$+ \frac{2}{\pi}\int_0^\infty dk(k\cos ky - K\sin ky) \int_{-\infty}^{\infty} d\nu\, B(k,\nu)\, \cosh\{x(\nu^2+k^2)^{1/2}\} e^{-i\nu z}, \tag{3.8}$$

because it is clearly an even function of x. From the boundary condition

$$\frac{\partial}{\partial x}(G_\infty(x) + G_{image}(x)) = 0 \text{ on } x = \ell$$

we can now find $A(\nu)$ and $B(k,\nu)$, using the Havelock expansions (3.5) and (3.6) for G_∞. We find that

$$(\nu^2-K^2)^{1/2} A(\nu)\{\sinh \ell(\nu^2-K^2)^{1/2}\} - 2Ke^{-Kf}\exp\{-\ell(\nu^2-K^2)^{1/2}\} = 0, \tag{3.9}$$

and

$$B(k,\nu)(\nu^2+k^2)^{1/2}\sinh\{\ell(\nu^2+k^2)^{1/2}\} - \frac{k\cos kf - K\sin kf}{K^2+k^2}\exp\{-\ell(\nu^2+k^2)^{1/2}\} = 0. \tag{3.10}$$

This completes the construction of the potential G_ℓ.

4 Expansion near the centre of the sphere

The integrals (3.7) and (3.8) are typically of the form

$$I(x,y,z) = \int A(v)\exp\{\xi(v)x + \eta(v)y + \zeta(v)z\}\, dv, \tag{4.1}$$

where the integrals may be single or double integrals, and where

$$\xi^2 + \eta^2 + \zeta^2 = 0,$$

since the integral satisfies Laplace's equation. In the integral (4.1) we write

$$\xi = i\eta(v)\cos\beta(v),\ \zeta = i\eta(v)\sin\beta(v) \text{ and } x = r\sin\theta\sin\alpha, y = f + r\cos\theta, z = r\sin\theta\cos\alpha. \tag{4.2}$$

Then

$$I(x,y,z) = \int A(v)\exp(\eta(v)f)\exp\left(\eta r\{\cos\theta + i\sin\theta\sin(\alpha+\beta(v))\}\right)\, dv. \tag{4.3}$$

In this integral we write

$$\exp\left(\eta r\{\cos\theta + i\sin\theta\sin(\alpha+\beta(v))\}\right) = \sum_{N=0}^{\infty}\frac{(\eta r)^N}{N!}\{\cos\theta + i\sin\theta\sin(\alpha+\beta(v))\}^N, \tag{4.4}$$

and in the series we use the known identity

$$
\begin{aligned}
&(\cos\theta + i\sin\theta\cos(\alpha+\beta))^N \\
&= \mathrm{P}_N(\cos\theta) \\
&+ 2\sum_{M=1}^{N}(-i)^M \frac{N!}{(N+M)!}\mathrm{P}_N^M(\cos\theta)(\cos M\alpha\cos M\beta - \sin M\alpha\sin M\beta). \qquad (4.5)
\end{aligned}
$$

This argument shows that the coefficient of $r^N\mathrm{P}_N^M(\cos\theta)\cos M\alpha$ in (2.2) involves the integral

$$\int A(v)\exp\{\eta(v)f\}\{\eta(v)\}^N\cos M\beta(v)\,dv.$$

In this way all the coefficients can be determined, and the boundary condition on the sphere can then be applied.

5 Discussion

There are still many aspects which we have not discussed. We mention only a few.

(1) The precise form of the coefficients $b(n,m:N,M)$ in equation (2.4) must be studied; these involve six parameters . Calculations show that these coefficients actually depend on fewer parameters. Similarly in two dimensions the parameters depend on three parameters, not on four.

(2) For $|z| > a$ there must be a modal expansion in each direction, of the form

$$(G_n^m)_\ell = \frac{1}{2}\Phi_n^m(y,z;0;\ell) + \sum_{s=1}^{\infty}\cos\frac{s\pi x}{\ell}\Phi_n^m(y,z;s;\ell), \qquad (5.1)$$

where the functions $\Phi_n^m(y,z;s;\ell)$ must be found. It has been shown that these functions have Havelock expansions in the y-direction and can be found either by deformation of contours or by Green's Theorem.

(3) The method can readily be extended to the general case where the centre of the sphere is not necessarily midway between the vertical walls.

(4) We have not discussed the expansion of G_∞ near the sphere. This presents little difficulty. We have already noted that the calculation of the expansion for the general multipole potential $(G_n^m)_\ell$ is similar to the calculation for the source potential, given above.

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On modeling nonlinear water waves

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1 Introduction

My interest in studying nonlinear dispersive long waves has an origin in the stimuli I received from Professor Georg Weinblum during my sabbatical visit in 1964-65 at the Schiffbau Instituet. Apparently, there had been in existence among a handful of the master experimentalists a puzzle that in conducting towing tank tests with ship models towed at transcritical velocities in shallow water, perplexing difficulties were invariably encountered in attempt to attain data with the ususal repeatability commonly known to their previous experience with noncritical cases. Subsequent studies later led to the interesting discoveries reported by Huang et al. (1982) and Wu & Wu (1982).

In modeling weakly nonlinear and weakly dispersive long waves, it has been a common practice to taking two key parameters, namely

$$\epsilon = h/\lambda \ , \qquad \alpha = a/h, \tag{1}$$

for characterising waves of typical length λ, amplitude a in water of undisturbed depth h. In this respect, it is so well said by Julian Cole (1968) that theories can be sought to show how different expansions based on different parametric regimes lead to different approximate equations.

In making attempts to explore and determine the basic mechanism underlying the remarkable phenomenon of periodic generation of upstream-radiating solitons by disturbances moving steadily at transcritical velocity, efforts have been devoted to examine the effects of theoreticl models of accuracy higher than that of Boussinesq's equations, as demonstrated by a previous study by Wu & Zhang (1996). To facilitate further studies, this work is an attempt to establish an exact model for describing propagation and generaton of nonlinear dispersive gravity-capillary waves of arbitrary amplitude on water of uniform depth. With this study, I wish to join my colleagues to commemorate the Centennial Celebration of Georg Weinblum.

2 The basic equations

Here we consider the class of three-dimensional long waves on a layer of water of uniform depth h, when undisturbed. The fluid moving with velocity $(\boldsymbol{u}, w) = (u, v, w)$ occupies the flow field in $-h \le z \le \zeta(\boldsymbol{r}, t)$, where $z = -h$ is a rigid horizontal bottom, $\zeta(\boldsymbol{r}, t)$ is the water surface elevation from the undisturbed plane at $z = 0$, measured at the horizontal position vector $\boldsymbol{r} = (x, y, 0)$ at time t, and $\boldsymbol{r}$ is unbounded, $|\boldsymbol{r}| < \infty$. Assuming the fluid incompressible, the velocity field irrotational, so the motion satisfies the Euler equations of continuity, horizontal and vertical momentum:

$$\nabla \cdot \boldsymbol{u} + w_z = 0, \tag{2}$$

$$\frac{d\boldsymbol{u}}{dt} = \boldsymbol{u}_t + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + w \boldsymbol{u}_z = -\frac{1}{\rho} \nabla p, \tag{3}$$

$$\frac{dw}{dt} = w_t + \boldsymbol{u} \cdot \nabla w + w w_z = -\frac{1}{\rho} p_z - g, \tag{4}$$

where $\nabla = (\partial_x, \partial_y, 0)$, ($\partial_x = \partial/\partial x$, etc.) is the horizontal projection of the vector gradient operator, p is the pressure, ρ the density and g the gravitational acceleration. Here, the subscripts t and z

denote differentiation. The boundary conditions are

$$w = D\zeta \qquad (D = \partial_t + \hat{\boldsymbol{u}}\cdot\nabla, \quad \text{on } z = \zeta(\boldsymbol{r},t)), \tag{5}$$

$$p = p_a(\boldsymbol{r},t) - \rho\gamma\nabla\cdot\boldsymbol{n} \qquad (z = \zeta(\boldsymbol{r},t)), \tag{6}$$

$$w = 0 \qquad (z = -h), \tag{7}$$

where $p_a(\boldsymbol{r},t)$ is a given external pressure disturbance gaged over the constant basic pressure (which is zero), $\rho\gamma$ is the uniform surface tension and $\boldsymbol{n}$ is the outward unit vector normal to the water surface.

The continuity equation (1) can be averaged over the water column $-h < z < \zeta$ under the kinematic boundary conditions (5) and (7), yielding the depth-mean continuity equation (Wu 1979, 1981),

$$\eta_t + \nabla\cdot(\eta\overline{\boldsymbol{u}}) = 0 \qquad (\eta = h + \zeta), \tag{8}$$

where the quantities with an overhead bar denote their depth-mean,

$$\overline{f}(\boldsymbol{r},t) = \frac{1}{\eta}\int_{-h}^{\zeta} f(\boldsymbol{r},z,t)dz \qquad (\eta = h+\zeta), \tag{9}$$

On the other hand, the horizontal momentum equation can be converted into an equation for $(\hat{\boldsymbol{u}},\zeta)$ where $\hat{\boldsymbol{u}}$ is the horizontal velocity at the water surface. For an arbitrary flow variable $f(\boldsymbol{r},z,t)$, it assumes its free surface value

$$f(\boldsymbol{r},\ \zeta(\boldsymbol{r},t),t) = \hat{f}(\boldsymbol{r},t), \tag{10}$$

say. Clearly, the rates of variation of these functions with respect to $\boldsymbol{r}$ and t satisfy the following relations

$$\partial_t\hat{f} = \partial_t f + \left.\frac{\partial f}{\partial z}\right|_\zeta \partial_t\zeta \qquad (z = \zeta), \tag{11}$$

$$\nabla\hat{f} = \nabla f + \left.\frac{\partial f}{\partial z}\right|_\zeta \nabla\zeta \qquad (z = \zeta). \tag{12}$$

¿From these fundamental relations it immediately follows that we have the theorem (see e.g. Choi 1995)

$$\left.\frac{df}{dt}\right|_{z=\zeta} = D\hat{f} \qquad (D = \partial_t + \hat{\boldsymbol{u}}\cdot\nabla) \tag{13}$$

Making use of these formulas, we readily deduce from (2)-(4) the result

$$D\hat{\boldsymbol{u}} + [g(t) + D^2\zeta]\nabla\zeta = -\frac{1}{\rho}\nabla p_a + \gamma\nabla\cdot\boldsymbol{n} \tag{14}$$

Here, we have extended our consideration to include the more general case of Faraday's waves produced in a horizontal water tank under resonant vibration, a case which is equivalent to having a time-dependent gravity acceleration with refenence to the tank frame. This resulting equation, though superficially involving only $(\hat{\boldsymbol{u}},\zeta)$, actually has incorporated the vertical momentum equation as well as the kinematic and dynamic conditions at the free surface to yield this equation of overall equilibrium. Furthermore, it is exact.

Thus, we have obtained two exact equations, one being the depth-mean continuity quation (8) for $(\overline{\boldsymbol{u}},\zeta)$, and the other the momentum equation (14) for $(\hat{\boldsymbol{u}},\zeta)$. This system of equations, however, is not closed because there are more unknown variables than the number of equations. Closure of the system can be achieved by further seeking the general solution to the field equation satisfied by the velocity potentail so as to provide an exact relation between the two sets of dependent variables, as will be shown below.

3 Nonlinear dispersive water wave models

Since the two new equations for the continuity and momentum are exact, we may ignore the nonlinearity parameter α by regarding it as arbitrary and consider first the special case of long waves by assuming the dispersion parameter $\epsilon = h/\lambda$ to be small. (It turns out that this assumption can also be eventually relaxed.)

Thus, with the vertical length scaled by h, horizontal length by λ, the three-dimensional Laplace equation satisfied by the velocity potential ϕ involves the parameter ϵ

$$\phi_{zz} + \epsilon^2 \nabla^2 \phi = 0 \qquad (-1 \leq z \leq \zeta). \tag{15}$$

Further, with ϕ scaled by $c\lambda$, where $c = \sqrt{gh}$ is the linear wave speed, ϕ satisfying (12) may assume an expansion of the form

$$\phi(\boldsymbol{r}, z, t; \alpha, \epsilon) = \alpha \sum_{n=0}^{\infty} \epsilon^{2n} \Phi_n(\boldsymbol{r}, z, t) = \alpha \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} [\epsilon(1+z)]^{2n} \nabla^{2n} \phi_0(\boldsymbol{r}, t; \epsilon). \tag{16}$$

Here, ϕ, jointly with the horizontal velocity $\boldsymbol{u}$ (scaled by c) and the elevation ζ (scaled by h) are assumed to be of order α, which is arbitrary. The function $\phi_0(\boldsymbol{r}, z, t; \epsilon)$, which is the only unknown involved in ϕ, may depend on the parameter ϵ resulting from appropriate regroupings of the complimentary solutions of the higher-order equations such that $\phi_0(\boldsymbol{r}, z, t; \epsilon) = O(1)$ as $\epsilon \to 0$. This regrouping is admissible provided the medium is uniform (h =const.) and unbounded, in the absence of any boundary effects of specific order in magnitude. From this expansion of ϕ, we deduce the horizontal and vertical velocity components, $\boldsymbol{u}$ and w, both scaled by c, from $\boldsymbol{u} = \nabla\phi$, $w = \epsilon^{-1}\partial\phi/\partial z$, giving

$$\boldsymbol{u} = \alpha \sum_{n=0}^{\infty} \epsilon^{2n} \boldsymbol{u}_n = \alpha \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} [\epsilon(1+z)]^{2n} \nabla^{2n+1} \phi_0(\boldsymbol{r}, t; \epsilon), \tag{17}$$

$$w = \alpha \sum_{n=1}^{\infty} \epsilon^{2n-1} w_n = \alpha \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)!} [\epsilon(1+z)]^{2n-1} \nabla^{2n} \phi_0(\boldsymbol{r}, t; \epsilon), \tag{18}$$

where $\boldsymbol{u}_0(\boldsymbol{r}, t) = \nabla\phi_0$. Now, the horizontal velocity at the bottom plane ($z = -1$) is simply

$$\alpha \boldsymbol{u}_0 = \alpha \nabla \phi_0. \tag{19}$$

We further have the depth-mean velocity $\overline{\boldsymbol{u}}$ and the on-surface velocity $\hat{\boldsymbol{u}}$ as

$$\overline{\boldsymbol{u}} = \alpha \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} [\epsilon(1+\zeta)]^{2n} \nabla^{2n} \boldsymbol{u}_0, \tag{20}$$

$$\hat{\boldsymbol{u}} = \alpha \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} [\epsilon(1+\zeta)]^{2n} \nabla^{2n} \boldsymbol{u}_0. \tag{21}$$

The present solution is of significance in drawing the conclusion that if $\boldsymbol{u}_o$ is analytic everywhere in the flow domain, the above series are all convergent within their radius of convergence, which is infinite. In such circumstances, the last two equations then define the functions

$$\overline{\boldsymbol{u}} = \overline{\boldsymbol{u}}(\boldsymbol{u}_o, \zeta) \qquad \text{and} \qquad \hat{\boldsymbol{u}} = \hat{\boldsymbol{u}}(\boldsymbol{u}_o, \zeta) \tag{22}$$

as analytic within the flow domain. Finally, from this result we may derive any one of the three basic sets of velocities, explicitly as a function of another by means of series inversion, the resulting series being noted to possess a finite radius of convergence.

In summary, we have now obtained three sets of models for describing nonlinear dispersive gravity-capillary waves on water of uniform depth in terms of the three sets of basic variables. In principle, these three models are equivalent in being exact for predicting this class of waves without limitation to the order of nonlinearity and dispersion, except that the fluid is taken to be incompressible and the flow, irrotational. For numerical computation based on these models, effective algorithms are being investigated. For the special case of nonlinear waves under small dispersive effects represented with the series truncated to some high orders, reference is made to Wu and Zhang (1996).

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