The excitation of waves in a very large floating flexible platform by short free-surface water waves.

Aad J. Hermans *

1 Introduction

At the eleventh workshop on water waves and floating bodies *Ohkusu* et.al. [2] described recent developments of the design of floating airports. These floating airports consist of a thin mat configuration of very large horizontal size. One must think of dimensions of about several kilometers by several hundred meters, while the thickness of the mat is several meters. For these configurations the natural bending rigidity is relatively small and the elastic deflection due to wave action will be dominant compared with the rigid body motions. This paper treats in principle the same problem as Ohkusu did but with a different mathematical method, a similar approach can be found in *Stoker* [5] for the motion of a floating elastic beam in shallow water. If one looks at the operational conditions of the airport it is expected that in general the deflections are generated by short waves, this means waves with a wavelength short with respect to the horizontal dimensions if the platform, but such that the thickness of the structure is small with respect to this wavelength. This motivates us to treat the mat as an infinite thin plate at the free-surface and to neglect its thickness. In this presentation results for the wave transmission and reflection by a half-plane and a strip will be shown. Finally the propagation of the disturbances due to an accelerating and a decelerating point source are shown, this can be seen as the simulation of the take-off and landing of an airplane.
The problem has some resemblance with the deflection of a floating ice plate. There is a lot of literature about this topic, hence, some more information can be distracted from these sources. The papers of *Schulkes* et. al. [3],[4] and *Meylan* et. al. [1] are mentioned for further reference.

2 Mathematical formulation

We consider the situation that the platform is positioned in an area where no tidal current is present and the incident waves are long crested. The waves will be incident with a arbitrary angle of incidence. To keep the formulae simple we treat the case of infinite water depth, it will be clear from the analysis that the case of finite water depth is a straight forward extension. Viscous effects are neglected as well. In the fluid domain we introduce the velocity potential $V(\boldsymbol{x},t) = \nabla\Phi(\boldsymbol{x},t)$. The incident wave will be written as,

$$\Phi_{inc}(\boldsymbol{x},t) = e^{ik(x\cos\alpha + y\sin\alpha)+kz-i\omega t}, \tag{1}$$

where $k = \omega^2/g$ is the dispersion relation and α the angle of incidence. The waves are short with respect to the length L and the beam B of the platform, i.e. $kB \gg 1$ and $L/B \gg 1$.

*Faculty of Applied Mathematics and Computer Science, TUDelft, Mekelweg 4, 2628 CD Delft, The Netherlands

The wave number k is be the proper large parameter in the asymptotic expansions. Usually in the application of the *ray method* the expansions are made with respect to k, in the final results it becomes evident what dimensionless parameter plays a dominant role. In our case it is expected that in beam seas the parameter kB is essential, while for pure head seas we have to consider kL. So we don't bother about the two large parameters. In the fluid domain we have the potential equation

$$\Delta\Phi = 0 \tag{2}$$

together with the linearized free-surface condition for (x, y) outside the platform

$$\frac{\partial^2\Phi}{\partial t^2} + g\frac{\partial\Phi}{\partial z} = 0 \quad \text{at } z = 0 \tag{3}$$

The platform is assumed to be a thin layer at the free-surface $z = 0$, this seems to be a good model for a shallow draft platform. The platform is modelled as an elastic plate with zero thickness. To describe the deflection w we apply the thin plate theory, this finally leads to an equation for Φ at $z = 0$ in the platform area

$$\left\{\frac{EI}{\rho g}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 + \frac{m}{\rho g}\frac{\partial^2}{\partial t^2} + 1\right\}\frac{\partial\Phi}{\partial z} + \frac{1}{g}\frac{\partial^2\Phi}{\partial t^2} = 0 \tag{4}$$

We now introduce harmonic waves in the form of the ray expansion

$$\Phi(\boldsymbol{x}, t; k) = a(\boldsymbol{x}, k)e^{ikS(\boldsymbol{x})-i\omega t} \quad \text{with} \quad a(\boldsymbol{x}, k) = \sum_{j=0}^{N}\frac{a_j(\boldsymbol{x})}{(ik)^j} + o((ik)^{-N}) \tag{5}$$

where $S(\boldsymbol{x})$ is the phase function and $a(\boldsymbol{x}, k)$ the amplitude function. Insertion of (5) into the Laplace equation (2) gives

$$-k^2\nabla_3 S\cdot\nabla_3 Sa + ik(2\nabla_3 a\cdot\nabla_3 S + a\Delta_3 S) + O(1) = 0 \tag{6}$$

The subscript 3 is used to indicate the three-dimensional ∇ and Δ operator. If no subscript is used the operators are two-dimensional in the horizontal plane. We compare orders of magnitude in (6). This leads to a set of equations for S and a_0 to be satisfied in the fluid region:

$$O(k^2) : \nabla_3 S\cdot\nabla_3 S = 0, \tag{7}$$

$$O(k^1) : 2\nabla_3 a_0\cdot\nabla_3 S + a_0\Delta_3 S = 0. \tag{8}$$

Next we insert (5) into the free-surface condition (3) outside the platform to get the following :

$$O(k^1) : iS_z = 1 \quad \text{and} \quad O(k^{-j}) : a_{jz} = 0 \text{ for } j = 0\cdots N \text{ at } z = 0 \tag{9}$$

The next step is to insert (5) into the condition at the platform. At this stage we have to make some estimates of the order of magnitude of the parameters of the platform. We introduce

$$\frac{EI}{\rho g} = \frac{\mathcal{E}}{k^4} \quad \text{and} \quad \frac{m}{\rho} = \frac{\mathcal{M}}{k}.$$

In this case the elastic properties and the mass of the platform play a role in the diffraction of the waves. The parameters $\mathcal{E}$ and $\mathcal{M}$ are of order one in k, this means they stay finite if k tends to infinity. This sounds like a contradiction, but it makes sense, it will be shown the values of these parameters may be large or small. The first two terms in the asymptotic evaluation of (4) become

$$O(k^1) : \left\{\mathcal{E}(S_x^2 + S_y^2)^2 - \mathcal{M} + 1\right\} iS_z = 1 \ \text{ at } z = 0 \tag{10}$$

and

$$O(k^0) : a_0 \left[\mathcal{E}\frac{\partial}{\partial z}(S_x^2 + S_y^2)^2 + 2S_z \left\{\frac{\partial}{\partial x}(S_x(S_x^2 + S_y^2)) + \frac{\partial}{\partial y}(S_y(S_x^2 + S_y^2))\right\}\right] + \tag{11}$$
$$a_{0z}\left\{\mathcal{E}(S_x^2 + S_y^2)^2 - \mathcal{M} + 1\right\} + 4a_{0x}S_xS_z(S_x^2 + S_y^2) + 4a_{0y}S_yS_z(S_x^2 + S_y^2) = 0$$

We combine the equations for the phase function at the free-surface with those in the water domain and obtain:

$$S_z = -i \ \text{ or } \ S_x^2 + S_y^2 = 1 \ \text{ outside the platform} \tag{12}$$
$$\left\{\mathcal{E}S_z^4 - \mathcal{M} + 1\right\} S_z = -i \ \text{ for } 0 < x < L \text{ and } 0 < y < B \tag{13}$$

Equation (13) has four solutions for $S_z : \{r_1, \pm r_2 + ir_3, \pm r_4 + ir_5\}$. Only the values of S_z with negative imaginary part are taken into account.

3 Infinitely long platform

We consider plane waves incident at $y = 0$. For convenience we assume the platform infinitely long, hence the waves are diffracted by a half-plane or a strip of width B. The wave field for $y < 0$ consists of an incident and reflected wave:

$$\Phi(\boldsymbol{x}, t) = e^{ik(x\cos\alpha + y\sin\alpha) + kz - i\omega t} + Re^{ik(x\cos\alpha - y\sin\alpha) + kz - i\omega t} \tag{14}$$

where R is the reflection coefficient.

Let us first solve the problem for the half-plane in other words with $B = \infty$. We then have (14) for $y < 0$ and for $y > 0$:

$$\Phi(\boldsymbol{x}, t) = \sum_{j=1}^{3} a_j e^{ik\left(x\cos\alpha + y\sqrt{n_j^2 - \cos^2\alpha}\right) + kir_jz - i\omega t} \tag{15}$$

where the square root is such it is positive or that its imaginary part is positive. This is to guarantee that the waves are either outgoing or evanescent. In this expression different types of solutions are combined. For all the components the amplitude function is a constant in the whole domain, in the case of a plane incident wave. For instance, for the real $n_1 < 1$ the solution consists, for angles of incidence with $|\cos\alpha| < n_1$ of a transmitted wave. This wave gives rise to reflections at $y = B$, if B is finite. If $|\cos\alpha| > n_1$ we have total reflection at $y = 0$, however the formulation (15) still gives a proper description of the evenescent mode. The angle for wich $|\cos\alpha_c| = n_1$ is called the critical angle α_c. The other two contributions are always of evenescent type, because the arguments of the sin and cos terms are always complex. The four unknown coefficients $\{R, a_j\}$ for $j = 1,2,3$ are completely determined by the boundary conditions at the edge of the platform. We require continuity of the wave elevation and its inclination, hence, we assume platform very flexible. We now obtain

$$\Phi|_{0-} = \Phi|_{0+} \text{ and } \Phi_y|_{0-}- = \Phi_y|_{0+} \text{ at } y = 0 \text{ and } z = 0, \tag{16}$$

furthermore at the edge of the platform we have the condition of zero moment and zero shear force

$$\frac{\partial^3}{\partial y^3}\left(\frac{\partial \Phi}{\partial z}\right) = \frac{\partial^2}{\partial y^2}\left(\frac{\partial \Phi}{\partial z}\right) = 0 \text{ at } y = 0+ \text{ and } z = 0 \tag{17}$$

We now solve four linear equations for the four unknown coefficients. In the figures 1 and 2 the elevation is shown for several values of the angle of incidence for $\mathcal{E} = 1$ and $\mathcal{M} = 0.5$. For the angle of incidence $\alpha = \pi/3$ we notice a plane wave propagating along the platform and two evanescent modes, while for the small value $\alpha = \pi/8$ total reflection occurs and the deflection of the platform consists of three evanescent modes.

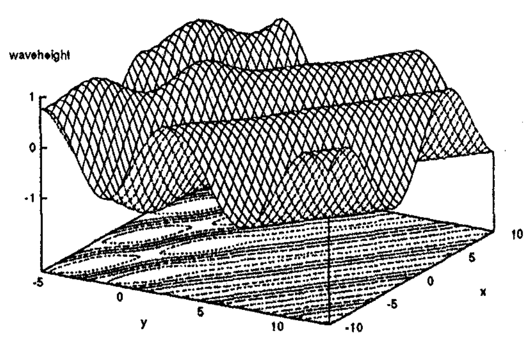

Figure 1: Waveheight for $\alpha = \pi/3$.

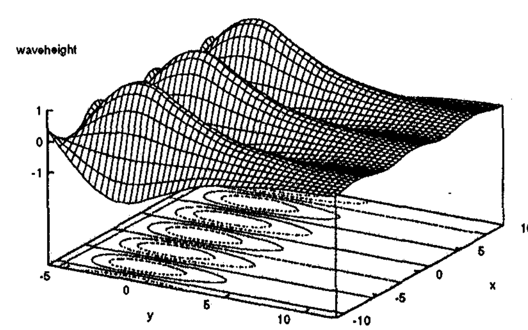

Figure 2: Waveheight for $\alpha = \pi/8$.

References

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[3] Schulkes, R.M.S.M., Hosking, R.J. and Sneyd, A.D., Waves due to a steady moving source on a floating ice plate. Part 2. *J.Fluid Mech.*, Vol. 180, pp. 297-318, 1987.

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