Numerical Method for Nonlinear Wave Loads on a Truncated Cylinder

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Introduction

The 'ringing' phenomenon has been observed recently in experiments. It occurs when the frequencies of resonant motions are several times higher than the wave frequencies and the wave amplitudes are of the same order as the characteristic length of the geometries, which indicate significant nonlinear effects.

A new nonlinear approach was introduced by Faltinsen *et al* (1994), referenced hereafter as FNV, and Newman (1994). A matched asymptotic expansion method is used to simplify the nonlinear effect. The analytical solution is carried out for a fixed infinitely-deep vertical cylinder, and the nonlinear loads are significant.

The present study is based on this nonlinear approach. In order to study practical structures such as a truncated cylinder or TLP, a numerical solution using the panel method is required. The numerical method is also necessary if one wants to consider the structure free to respond to the waves, and the effect of the finite fluid depth. The proposed method will encompass all of these generalizations.

Some preliminary numerical results for a truncated cylinder with regular waves are presented in this abstract, and the results are compared with the analytical solution given by FNV.

Formulation

In FNV, it is assumed that

$$A/a = O(1), \qquad \text{and} \qquad KA << 1, \quad Ka << 1, \tag{1}$$

where A is the wave amplitude, a the radius of the cylinder, and K is the wavenumber.

The total potential ϕ in the inner domain close to a fixed cylinder is

$$\phi \;=\; \phi_D + \psi, \tag{2}$$

where ϕ_D is the linear diffraction potential, and ψ is the nonlinear potential.

The leading order boundary conditions for ψ are

$$\psi_R = 0, \qquad \text{on } R = 1, \tag{3}$$

$$\begin{aligned}\psi_Z &= -\frac{g}{a}(-2\nabla\phi_D \cdot \nabla\phi_{Dt} - \frac{1}{2}\nabla\phi_D \cdot \nabla(\nabla\phi_D)^2) \\ &= f(\mathbf{x}, t), \qquad \text{on } Z = 0,\end{aligned} \tag{4}$$

where $R = r/a, Z = (-z + A\sin(\omega t))/a$, and g denotes the gravity. The horizontal plane $Z(t) = 0$ coincides with the intersection of the incident wave with the body axis, which is moving up and down with time. The free surface condition for ψ is an inhomogeneous Neumann condition. The forcing function f is defined on the plane $z = 0$, while equation (4) has to be satisfied on $Z(t) = 0$.

Applying Green's Theorem, the boundary integral equation for ψ is

$$2\pi\psi(\mathbf{x}, t) + \iint_{S_b(t)} \psi(\xi, t)\frac{\partial G(\mathbf{x}, \xi, t)}{\partial n}d\xi = \iint_{S_Z(t)} f(\xi, t)G(\mathbf{x}, \xi, t)d\xi, \quad \mathbf{x} \in S_b(t), \tag{5}$$

where $S_b(t)$ is the submerged body surface below $S_Z(t)$, the portion of the plane $Z(t) = 0$ exterior to the cylinder. The Green function is defined as

$$G(\mathbf{x}, \xi, t) = \frac{1}{r} + \frac{1}{r'}, \tag{6}$$

where the image source is above $S_Z(t)$. Since $\partial G(\mathbf{x}, \xi, t)/\partial n = 0$ on $S_Z(t)$, there is no integral over $S_Z(t)$ for the unknown ψ on the left side of equation (5). Both $S_b(t)$ and $S_Z(t)$ are changing with time, so the solution for ψ must be evaluated in the time domain.

Numerical Method

The gradient of the first order potential on the free surface is needed to evaluate the forcing function f. This is evaluated using the three dimensional low-order panel code TiMIT, which has been developed for linearized analysis of radiation and diffraction problems in the time domain (Bingham *et al* 1994). The source formulation using the transient free-surface Green function is used for this problem to calculate the source strength on the body.

Because of the long wave approximation, the computation time is reduced by eliminating the convolution over the previous time history and using the simplified Green function (6) to calculate the diffracted velocities on the free surface from the source distribution on the body. Central differencing is used to calculate the derivatives of the velocities with respect to time and space. The forcing function f is evaluated on the free surface along the normal to the body boundary at the centroid of each waterline panel.

The numerical solution of f at a typical time is shown in Figure 1. The solution is not correct within a distance of about half a waterline panel length from the body (Zhao

& Faltinsen 1989). For a deep cylinder, the solution is expected to compare well with the analytical solution. Figure 1 demonstrates that the forcing function f tends to zero as the field point moves away from the cylinder and confirms that the forcing function is a localized function. Figure 1 also shows that the asymptotic method is valid for $Ka < 0.1$.

Equation (5) is solved by a seperate low-order panel code. Considering the property of f, the free surface integral is truncated at a finiate circle with radius $r = b$. The free surface is discretized into planar quadrilateral panels, and the integrand is evaluated at the centroid of each panel. To keep the same number of panels on the body and to preserve the uniformity of the panels, the vertical coordinates of each panel on the body are stretched corresponding to $Z(t) = 0$ at each time step. At each time step, the right hand side and the left hand side of (5) are reevaluated to solve for ψ. In FNV, ψ is written in the Fourier series,

$$\psi = \sum_{m=0}^{3} c_m(t)\psi_m(R, Z)\cos m\theta, \tag{7}$$

where $c_0 = c_2 = \omega K A^2 a \sin 2\omega t$, and $c_1 = c_3 = \omega K A^3 a \sin^3 \omega t$. To facilitate the comparsion with FNV, numerical Fourier transformed values of $\psi_1(1, Z)$ and $\psi_2(1, Z)$ are shown in Figure 2. The numerical solution converges to the analytical solution as the radius b and the number of panels on the free surface are increased. The radius of b only needs to be 2 or 3 times larger than a to achieve the converged solution. ψ decays rapidly as Z increases.

References

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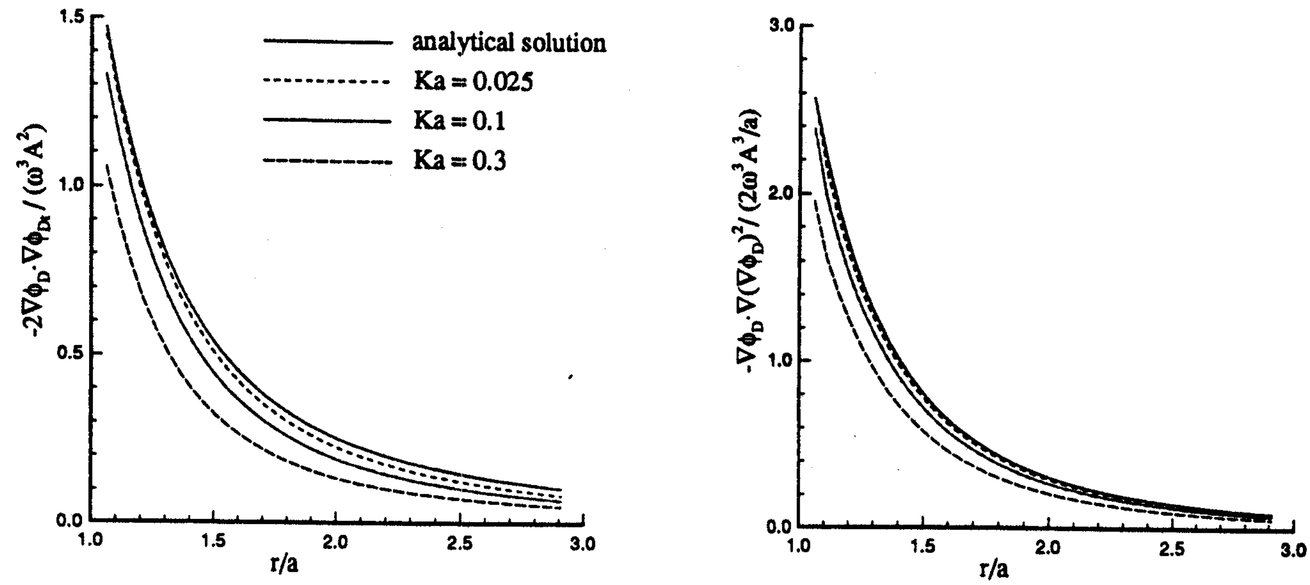

Figure 1: The first and second terms of the right hand side of equation (4) as functions of r/a for the cylinder $a/T = 1/8$. T is the draft of the cylinder. The number of panels on the cylinder is 576.

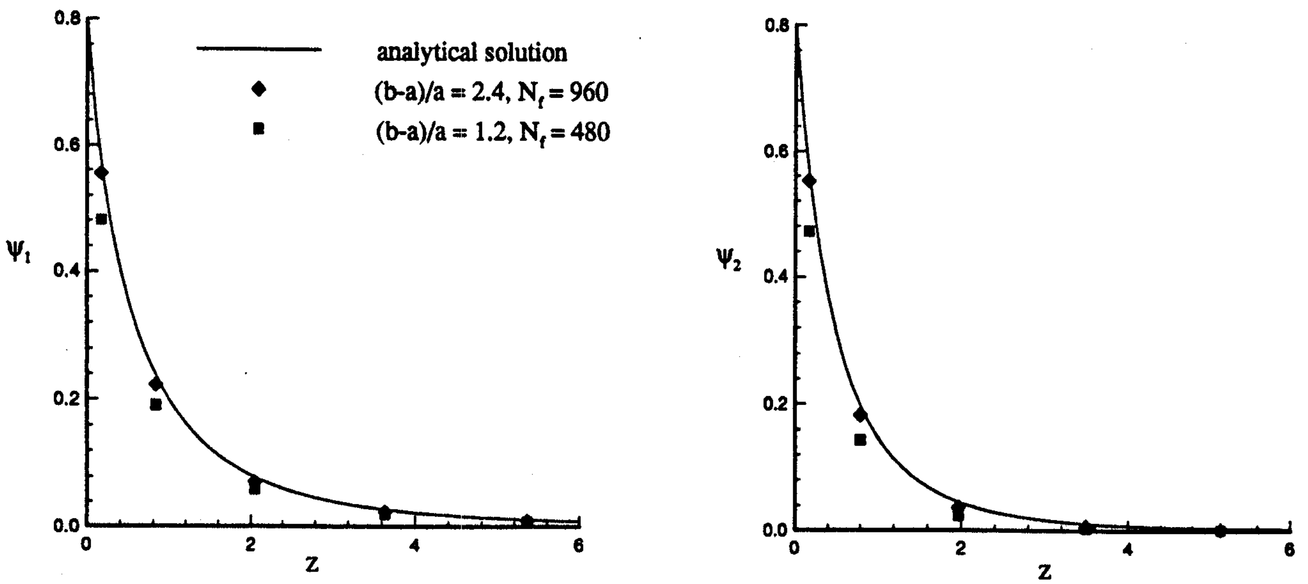

Figure 2: Fourier components of ψ as functions of Z. The geometry and the number of panels are defined in Figure 1. $Ka = 0.025$. N_f is the number of panels on $S_Z(t)$.

DISCUSSION

Rainey, R. C. T.: A T.L.P. in large waves moves horizontally by a substantial amount, multiplying the relative velocities and accelerations by a factor F, where $F \approx 0.1$ for a small one. Referring to my paper at this workshop, the effect is to multiply my "oblique slam" force by F^2 , and my "surface distortion" force by F^3 . I believe their sum corresponds to the force caused by your nonlinear potential Ψ - so I would be very interested to know how much that changes when you similarly move your cylinder horizontally in phase with the water particles.

Zhu, X.: I am sorry, we haven't done any calculations about a cylinder moving horizontally.