Fully-Nonlinear Three-Dimensional Interaction between Water Waves and a Surface-Piercing Body

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INTRODUCTION

Recent successes in the numerical simulation of fully-nonlinear free surface flows in three dimensions (*e.g.*, Xü & Yue 1992) has opened the way for the solution of meaningful nonlinear fluid-structure problems in marine applications. The direct extension of these techniques for practical applications, however, still represents a significant challenge. The complications are primarily associated with the proper treatment of the intersections of surface-piercing bodies with waves, the need for robust moving grid handling, the appreciable computational costs associated with large nonlinear domains, and, for the diffraction problem in particular, the incompatibility of the prescribed initial condition for the (nonlinear) ambient waves with the (no flux) boundary condition on the body.

We consider the numerical simulation of fully-nonlinear water waves interacting with surface-piercing bodies. To demonstrate the present capabilities, we show computational results for two nonlinear wave-body problems: (i) the diffraction of steep incident waves by a vertical cylinder; and (ii) the radiation of waves by the large-amplitude oscillations of a floating body. In addition to fundamental interest in the nonlinear forces and wave patterns, our study of these model problems also serves to illustrate the solution to the aforementioned technical difficulties.

NUMERICAL METHOD

We pose an initial-boundary-value problem for the scalar velocity potential of an ideal and irrotational flow. The potential satisfies Laplace's equation in the field, Neumann conditions on instantaneous body (and bottom) boundaries, and exact kinematic and dynamic conditions on the free surface. We use a mixed-Eulerian-Lagrangian (MEL) scheme which integrates the free surface conditions in time explicitly by following Lagrangian points on that surface. The method is thus not restricted to free surface elevations describable by single-valued functions. At each time step, the Eulerian boundary-value problem for the potential is solved using a boundary-integral-equation (BIE) formulation. Following Xü & Yue (1992), we apply a direct (Green's identity) formulation and employ high-order (quadratic) isoparametric boundary element method (QBEM) for the solution of the BIE. The accuracy and efficiency of the method has been demonstrated, for example, in their study of the kinematics of steep overturning three-dimensional waves.

The present work describes the extension of this nonlinear three-dimensional capability to problems involving surface-piercing bodies and our attempts at obtaining practically useful nonlinear predictions. Some of the major areas of improvement and development include:

Modification of the double periodic Green function

A satisfactory treatment of the far-field closure is a fundamental question for nonlinear wave-body problems in unbounded domain. Perhaps the most simple (theoretically but not necessarily numerically) solution is to impose periodic condition. In Xü & Yue (1992)'s original formulation for deep water with double periodic boundary condition, the Green function is taken as the double summation of the infinite series of source. This choice of the Green function results in a non-trivial unknown constant, which is proportional to the potential at infinite depth and in principle varies in time, in the (Green's identity) BIE. This unknown is solved as part of the linear system by an introduction of the Gauss condition, which essentially stipulates that there should be no net flux at infinite depth. We make a simple modification to the Green function by subtracting the leading behavior at infinite depth of the original Green function. This eliminates the extra undetermined constant from the BIE. Significantly, we found that this also reduces the required number of iterations for solving the resulting system by a factor of O(10).

Far-field closure using linear matching

Another treatment of the far-field radiation condition is by the matching technique. In three dimensions, the energy density of diffracted and radiated waves must in principle decrease inversely with radial distance. Thus, at some given radius selected on the basis of wave amplitude, a matching of the nonlinear inner solution to a general linearized wavefield should in theory allow nonlinear simulations to be carried out indefinitely in time. This kind of linear matching closure was demonstrated for the case of (vertically) axisymmetric flows by Dommermuth & Yue (1987a) who used a fixed matching boundary.

We extend such a matching to general three-dimensional problems using (transient) linearized wave Green functions and quadratic boundary elements. Furthermore, to minimize the nonlinear inner domain at any time, we incorporate a so-called body nonlinear treatment (*e.g.*, Lin & Yue 1990) so that the matching conditions are applied on a moving surface. The resulting integral equation for the linearized outer solution now involves convolution integrals over the matching boundary as well as a waterline integral along the intersection with the free surface. The nonlinear domain is minimized by translating and deforming the matching boundary according to local steepness of the waves. One of the major requirements and difficulties of such an approach is the need for robust regridding of the Lagrangian (inner free-surface) and Eulerian (matching) surfaces and their intersection.

Treatment of wave-body intersections

In the context of the MEL approach, where the solution must be integrated in time at every node, the relevant error measure is the *maximum* error. For this purpose, constant panel method (CPM) turns out to be inadequate (*cf.*, Xü & Yue 1992) since for mixed boundary-value problems CPM converges non-uniformly typically for the maximum error on the Dirichlet side of the Neumann-Dirichlet intersection line. This difficulty is effectively removed when higher order BIE panels are used. One of the lowest order element that is still efficacious is QBEM. In addition to the expected approximately cubic maximum error convergence with panel size, QBEM, which has nodes on the panel boundaries allows for robust double-node treatment at the body-free surface (Neumann-Dirichlet) intersections.

Regridding used in combination with MEL

When applying MEL, fluid particles on the free surface may become clustered in regions of high gradients in the flow field quantities. This time evolving deformation of the Lagrangian grid in general result in the numerical degradation and eventual breakdown in the solution of the BIE as panels become ever more distorted. The accurate and robust optimal regridding based on the flow characteristics is important and often essential to ensure a successful long time simulation.

Specification of initial condition for diffraction problem

In the context of a fully-nonlinear simulation, the specification and characterization of the (non-linear) incident wave is critical to the success of the diffraction problem. We start with the *exact* Stokes' wave that may be modulated (*e.g.*, Dommermuth & Yue, 1987b), which is particularly suited to periodic boundary condition. Another problem with the diffraction problem is that the prescribed initial condition for the ambient waves would not in general be compatible with the zero-flux body boundary condition. In the absence of special treatment, incident waves of moderate steepness would induce large error at the body intersection and can cause the simulation to break down immediately. Several treatments are proposed and investigated, among which are positioning of the strut according to the initial field velocity of incident wave, modulation/modification of the initial free surface potential near the strut, applying localized pressure to suppress initial jetting around the intersection, and the implementation of body boundaries which have deformability and/or permeability that vary slowly to zero in time.

Capability to treat periodic incident wave with current

Diffraction or radiation of waves in the presence of *current* is of science interest and engineering importance. An interesting problem arises in the context of periodic boundary conditions as to whether the body or its forced motions can in time change the magnitude of the current. This question has direct computational implications since our Green's identity BIE in general requires periodicity not only of the velocity but also the potential. We are able to show that, for a simply-connected fluid domain, the non-periodic part (*i.e.*, the mean current) of the total potential is time invariant regardless of the existence of bodies or their motions. Thus the mean component of the potential can be subtracted out (to yield a periodic potential) from the BIE and re-enters the initial-boundary-value problem only in the time integration of the free-surface boundary conditions.

RESULTS

The validity of the method is first confirmed by a systematic series of tests for convergence and conservation of mass and energy. We focus our interest on two applications:

(1) The nonlinear forces and waves associated with the diffraction of steep Stokes' wave by

a surface-piercing cylinder. Recent field and laboratory observations in the context of large offshore structures suggest that nonlinear diffraction mechanisms may be responsible for third and higher harmonic excitations. The magnitudes of these excitations can be appreciable and indeed critical in terms of resonant amplifications of the wave loads. The present computation aims to provide a much needed confirmation of such nonlinear mechanisms.

(2) The nonlinear radiation problem associated with the forced oscillation of a floating body. A problem of fundamental interest is the observation of modulated radial waves from an axisymmetric body in heaving motion (*e.g.*, a heaving sphere, Tatsuno *et al* 1969). Recent analytic predictions based on a cross-wave model (Becker & Miles, 1992) under-estimate the wave amplitude by an order of magnitude suggesting the possible importance of nonlinear effects not accounted for in the analysis. The resolution of this fundamental problem requires a nonlinear three-dimensional wave-body capability and is a natural candidate for the present code (with a matching far-field boundary).

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