Nonlinear Wave Loading on a Floating Body

G.X. Wu+, Q.W. MA+ and R. Eatock Taylor++

+ Department of Mechanical Engineering, University College London, Torrington Place, London WC1E 7JE, U.K.

++ Department of Engineering Science, University of Oxford, Parks Road, Oxford OX1 3PJ, U.K.

Nonlinear wave loading is usually solved by the time stepping method. Once the potential is found at each time step, the force on the body is obtained by integrating the pressure obtained from the Bernoulli equation over the body surface. The difficulty, however, is the term $\partial\phi/\partial t$. Several methods have been used in various publications. Lin, Newman and Yue (1984) for example obtained $\partial\phi/\partial t$ by calculating $\mathrm{d}\phi/\mathrm{d}t$. But this method has several limitations. One of them is that to calculate $\mathrm{d}\phi/\mathrm{d}t$ the same fluid particle has to be followed. This is particularly problematic when remeshing is applied.

An alternative has been adopted by Cointe et al (1990) and Cao, Beck & Schultz (1994). They obtained $\partial\phi/\partial t$ by solving a boundary value problem which is similar to that for the potential itself. The difficulty with this method is that it requires the acceleration of the body as part of its body surface boundary condition, which in turn requires the pressure and therefore $\partial\phi/\partial t$. This suggests that iterations may be required for a floating body. Another technique is to combine the boundary value problem for $\partial\phi/\partial t$ with the equation of motion to form an integral equation (Van Daalen 1993).

Recently Wu & Eatock Taylor (1994a,b) have adopted the finite element formulation to solve the two dimensional nonlinear problem in the time domain. The following equation is used to calculate the force on a submerged cylinder

$$F = -\rho\frac{\mathrm{d}}{\mathrm{d}t}\int_{S_0}\phi n \mathrm{d}S + \rho\int_{S_0}\left(\frac{\partial\phi}{\partial n}\nabla\phi - \frac{1}{2}\nabla\phi\nabla\phi n\right)\mathrm{d}S \tag{1}$$

where ρ is the density of the fluid, and S_0 is the body surface and $\mathbf{n}$ is its normal pointing out of the fluid domain. The equation is similar to that derived by Newman (1977) in the unbounded fluid domain. Its advantage is that the derivative with respect to time in the first term is far easier to calculate than the derivative of ϕ itself. The finite element method used by Wu & Eatock Taylor is essentially to solve the following equation

$$\int_R \nabla N_i \sum_{j=1}^{n}\phi_i\nabla N_j \mathrm{d}R|_{j\notin S_F} = \int_{S_R} N_i f \mathrm{d}S - \int_R \nabla N_i\sum_{j=1}^{n}\phi_j\nabla N_j\mathrm{d}R|_{j\in S_F} \quad i\notin S_F \tag{2}$$

where N_i is the shape function, R is the fluid domain, n is the number of elements, S_F is the free surface, S_R is the rigid body surface and f is the boundary condition (i.e $\partial\phi/\partial n=f$). In the program, linear shape functions are used together with triangular elements. This allows the left hand side of the equation to be calculated from the areas of the elements. The method has further adopted optimisation to minimise the band width, and made use of symmetry of the matrix. All these have significantly improved the CPU and memory requirement. Figure 1a and 1b give some of the calculated results. They correspond to a circular cylinder submerged at h=1.5a and undergoing forced horizontal motion governed by $\delta=\delta_0(1-\cos\omega t)$ with δ_0/a=0.1 and $\omega\sqrt{(a/g)}$=1.0. The water depth has been taken as d=4a. The results from the finite element method (FEM) have been compared with those obtained from the boundary element method (BEM) and excellent agreement can be seen from the figures. It is particularly interesting to see that the numerical results support the conclusion of Wu (1993) regarding the force component on a symmetrical body oscillating horizontally.

Wu & Ma (1994) have further used the above formulation to analyse the three dimensional problem. Linear shape functions are used together with tetrahedral elements. In particularly, they have derived the following equation to calculate the force on a three dimensional floating body

$$F = -\rho \frac{\mathrm{d}}{\mathrm{d}t} \int_{S_0} \phi \boldsymbol{n} \mathrm{d}S - \rho \int_{S_0} gz\boldsymbol{n}\mathrm{d}S + \rho \int_{S_0} \left(\frac{\partial \phi}{\partial n} \nabla \phi - \frac{1}{2} \nabla\phi\nabla\phi \boldsymbol{n} \right) \mathrm{d}S + \rho \oint_{C_0} \phi \frac{\partial \phi}{\partial l_0} \boldsymbol{n} \mathrm{d}C \tag{3}$$

where C_0 is the waterline of the body and l_0 is in the direction perpendicular both to **n** and C_0. They have also found that as an alternative the force can be calculated by the following equation

$$\begin{aligned} \boldsymbol{F} = & -\rho \frac{\mathrm{d}}{\mathrm{d}t} \int_{S_0+S_F} \phi \boldsymbol{n} \mathrm{d}S - \rho \int_{S_0+S_F} gz\boldsymbol{n}\mathrm{d}S + \frac{\rho}{2} \int_{S_B} \nabla\phi\nabla\phi \boldsymbol{n}\mathrm{d}S \\ & -\rho \int_{S_\infty} \left(\frac{\partial \phi_i}{\partial n} \nabla \phi_i - \frac{1}{2} \nabla\phi_i\nabla\phi_i \boldsymbol{n} \right) \mathrm{d}S - \rho \oint_{C_\infty} \phi_i \frac{\partial \phi_i}{\partial l} \boldsymbol{n} \mathrm{d}C \end{aligned} \tag{4}$$

where S_B is the sea bed, S_∞ is a surface at infinity and C_∞ is its waterline. ϕ_I in this equation is the incident potential.

Here we use equations (3) and (4) calculate the force on a surface-piercing vertical circular cylinder of radius R_0. It is fixed on the see bed. The initial wave elevation is assumed as

$$\varsigma(x,y,0) = \varsigma_0 \exp(-k\,|\,x\,| - k\,|\,y\,|) \sin kx \tag{5}$$

in a Cartesian system whose origin is at the centre of the cylinder and the undisturbed the free surface, and z points upwards. The potential on the free surface is assumed to be zero when t=0. For the result provided in the figure 2, the computational domain is truncated at R=20a where a rigid body boundary condition is imposed. It can be seen that both equations (3) and (4) give indistinguishable results. It should be noticed, however, that equation (4) does not need one to calculate the derivatives of the potential with respect x, y and z on the body surface. It can be further simplified when there is no incident potential at infinity. The third term will also disappear if the water depth is infinite and for the horizontal components if there is a flat bottom. But this equation contains an integration over the entire free surface. It is therefore more likely to be affected by the error at the truncated boundary.

Results in figures 1 and 2 show that equations (2), (3) and (4) are effective for calculating the force. The drawback is that they do not provide the detailed pressure distribution. The direct calculation of $\partial\phi/\partial t$ will provide such information. But as discussed at the beginning, this method also has several drawbacks in terms of CPU and accuracy. On the other hand, error in calculation of pressure will lead to error in acceleration of the body which will further affect the velocity and position of the body at the next time step. As the time step increases, the accumulated error may be coupled with that on the free surface to cause inaccuracy, or even instability.

What we propose therefore is to use equations (3) or (4) to calculate the force. Newton's law will then provide the acceleration. If the pressure distribution is required at a particular time step, we can use the acceleration obtained in this way as the body surface boundary condition for $\partial\phi/\partial t$. We are now implementing this method in our program for the three dimensional problem. At this stage, we use the information from the previous time step as the boundary conditions for $\partial\phi/\partial t$. By integrating the pressure obtained this way, we obtain the force on the body. Calculation is made for the vertical cylinder and results are given in figure 2. They are in good agreement with those obtained from equations (3) and (4). But it should be pointed out that the cylinder is fixed in this case and agreement is easier to obtain. The problem will be far more complicated if the body is free to move. Results for these cases will be provided at the workshop.

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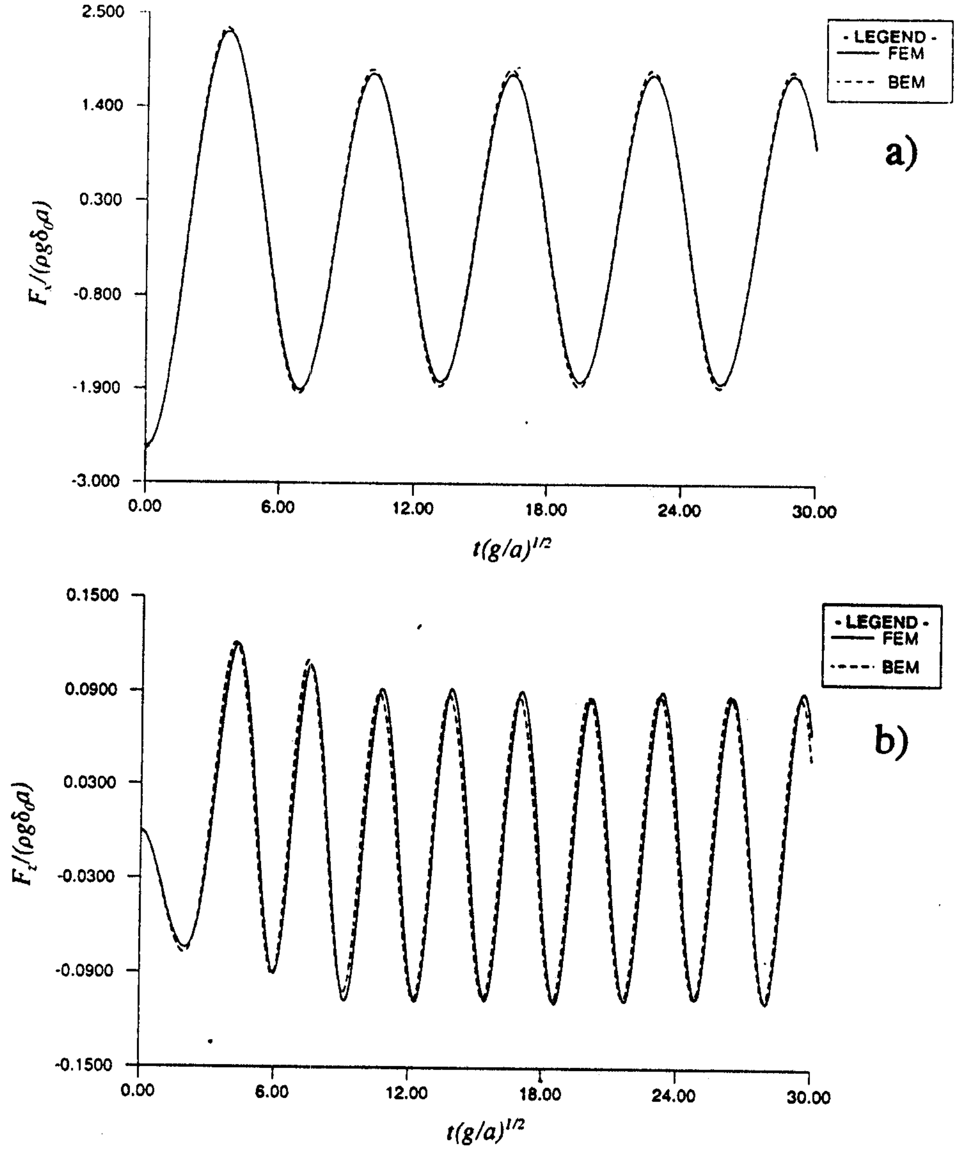

Figure 1 Forces on a submerged circular cylinder undergoing horizontal motions a) F_x ; b) F_z

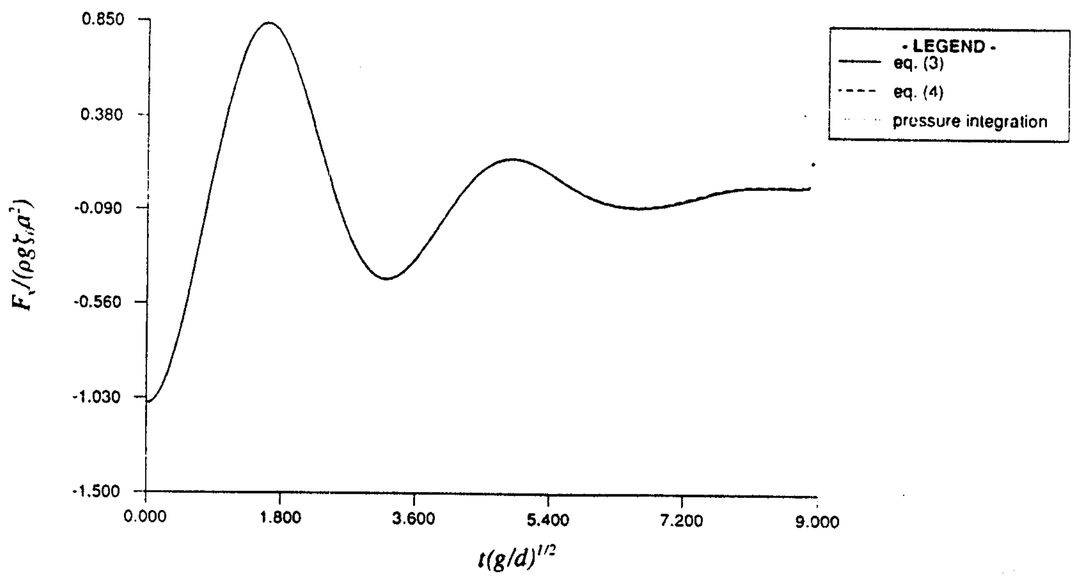

Figure 2 Horizontal force F_x on a vertical circular cylinder (a/d = 0.2, ζ_0/d = 0.1)