Acoustic effects on water impact

Alexander Korobkin
Lavrentyev Institute of Hydrodynamics, 630090, Novosibirsk, Russia

1. Introduction

Impact of the bottom of ships in waves is called slamming. The slamming causes hull vibration, structural damage of the bottom and leads to speed loss. The initial stage of the impact is of particular interest. Experimental studies indicate that the hydrodynamic loads are maximum at rather small times. That is why the analysis of the initial stage gives helpful estimates for the dynamic-strength investigations and provides the initial data for further numerical calculation.

The ideal incompressible liquid model is usually used under both numerical and analytical studies of the water impact problem. However, at the initial stage of the impact the hydrodynamic load and the flow structure are changed very sharply. In some cases these changes are of shock character, and the liquid compressibility has to be taken into account. Usually the duration of the initial stage when the compressibility effects are of importance is quite small. This duration depends on the bottom shape and the Mach number $M = V/c_0$ where V is the impact velocity, c_0 is the sound velocity in the water at rest. The sound velocity is about $1500 m/sec$, but the real impact velocities in ship hydrodynamics are relatively small, they are of order of some meters per second. This means that the variation of the liquid density under the impact is negligible, and the sound velocity may be approximately taken as the constant c_0. Moreover, at the initial stage for small Mach numbers the equations of the liquid motion, as well as the boundary conditions can be linearized, and the boundary conditions can be put at the initial position of the liquid boundary. In this case we obtain so called acoustic theory of the water impact.

The acoustic theory allows us to get more information about details of the water impact and to understand the processes occured. This theory is able to describe cavitation under the impact, formation of high-speed spray jets, the pressure distribution over the wetted part of the bottom, as well as to explain the high hydrodynamic loads and loss of the energy under the impact.

The goal of the present paper is to demonstrate a role of acoustic effects on water impact. These effects may be of importance in situations when the incompressible liquid model fails. This occurs in the following two cases: 1. accelerations of the liquid particles are high; 2. velocity of the involving the liquid particles in motion is high. The first case is typical one for the impact on a floating body (Section 2). A blunt-body impact illustrates the second case (Section 3). The speed of the expansion of the wetted part of the blunt-body bottom is infinite at the initial moment and decreases thereafter. Infinite speed of the contact region expansion is possible for a pointed body of a special geometry as well (Section 4). This case is similar to the bow flare slamming and may be typical for an elastic bottom.

2. Energy distribution under water impact

The impact on a half-sphere floating on a liquid free surface is considered. The origin of the Cartesian coordinate system $x'y'z'$ is taken at the centre of the sphere. Initially the liquid is at rest and occupies a domain $z' > 0$, $r' = \sqrt{x'^2 + y'^2 + z'^2} > R$, where R is the sphere radius. A region $z' = 0$, $r'_1 = \sqrt{x'^2 + y'^2} > R$ corresponds to the initial position of the free surface. Then the body starts to penetrate the liquid vertically at a constant velosity V. We take R as the lengthscale, the quantity R/c_0 as the timescale, the impact velocity V as the scale of the liquid velocity. Non-dimensional variables, which are designated by the above-mentioned terms without a prime, are used in this Section.

Within the framework of the acoustic approximation which is valid when $M = V/c_0 \ll 1$ the liquid flow is described by the velocity potential $\Phi(r, \theta, t)$ where $\cos\theta = z/r_1$. The potential satisfies the wave equation in the region $0 \le \theta \le \frac{\pi}{2}$, $r > 1$, the boundary conditions on the free surface: $\Phi = 0$ where $\theta = \frac{\pi}{2}$, $r > 1$, on the solid surface: $\partial\Phi/\partial r = \cos\theta$ where $r = 1$, $0 \le \theta < \frac{\pi}{2}$, and the initial conditions: $\Phi = \partial\Phi/\partial t = 0$ at $t = 0$. When the solution of the problem has been found, the pressure $p(r, \theta, t)$ in the liquid is determined by the linearized Cauchy-Lagrange integral $p = -\Phi_t(r, \theta, t)$. In particular, $p(1, \theta, t) = e^{-t}\cos t \cos\theta$ on the entering surface. It is seen that the pressure $p(1, \theta, t)$ is zero on the contact line, is negative when $\frac{\pi}{2} + 2\pi k < t < \frac{3\pi}{2} + 2\pi k$, $k = 1, 2, ...$, and decays exponentially as $t \to \infty$.

Internal energy of the compressed liquid Π' is given in the acoustic approximation by

$$\Pi' = \frac{1}{2\rho_0 c_0^2} \int\int\int p'^2 dx'dy'dz'$$

where ρ_0 is the liquid density, and the integration is carried out over the region of the liquid flow. Analytical calculations provide the formulae for both the internal Π' and the kinetic T' energy of the flow

$$\Pi' = m_a V^2 (\frac{1}{4} - e^{-2t} f_2(t)), \quad T' = m_a V^2 (\frac{3}{4} + e^{-t} f_1(t) + e^{-2t} f_2(t)),$$

as well as for the external energy A' that has to be applied to the body to move it at the constant velocity

$$A' = m_a V^2 (1 + e^{-t} f_1(t))$$

where $f_1 = \sin t - \cos t$, $f_2 = \frac{1}{4}(\cos 2t - \sin 2t)$, $m_a = \frac{1}{3}\pi\rho_0 R^3$ is the added mass of the semi-submerged sphere. It is worth noting that the theory of 'impulsive pressure' usually used in ship hydrodynamics predicts

$$T'_{inc} = \frac{1}{2} m_a V^2, \quad A'_{inc} = m_a V^2.$$

Comparing the results obtained for the acoustic theory and for the incompressible liquid model, we can say that 1) the quadratic terms in the Cauchy-Lagrange integral can be neglected at the initial stage when $c_0 t'/R = o(\ln M^{-1})$; 2) the 'impulsive pressure' theory predicts that only a half of the external energy transfers to the kinetic energy of the liquid motion, it is shown here that another half of the external energy is taken away by the acoustic wave; 3) a quarter of the external energy transfers to the internal energy of the compressed liquid; 4) one third of the total kinetic energy is localized near the shock front, and two third of it is the kinetic energy of the main flow region.

3. Duration of the acoustic stage

In order to obtain realistic results at the initial stage of a blunt-body impact on a liquid surface, the acoustic effects have to be taken into account. For a blunt body there is an instant t'_* such that the free surface remains undisturbed when $0 < t < t'_*$, and the shock front is attached obliquely to the contact points. Then the shock front escapes on the free surface, sharply deforming it. There is no doubt that for large times when the shock wave is away from the body the pressure distribution and the flow near the entering bottom are given by the Wagner theory. But the rate of convergence of the acoustic solution to the Wagner one is small. To demonstrate this point, let us consider a model problem which one may connect with a crane operation in dock.

Initially the liquid is at rest and occupies a lower half-plane $y' < 0$, and a blunt body touches its boundary $y' = 0$. Left the body the liquid is covered with a rigid half-infinite plate the right-hand edge of which is taken as the origin of the Cartesian coordinate system $x'Oy'$. The liquid boundary right the body $x' > 0$ is free. Then the body starts to penetrate the liquid with a constant velocity V. We shall determine the liquid flow and the pressure distribution. There is a single contact point between the free surface of the liquid and the entering surface. That is why this problem is much simpler than the general one with two contact points positions of which are unknown in advance. This makes it possible to investigate the acoustic solution carefully and get some estimates of the compressibility influence on both the flow structure and the integral characteristics of the impact.

Non-dimensional variables are used below. They are chosen so that after scaling both the sound velocity in the liquid at rest and the impact velocity are equal to unity in the new variables. The boundary-value problem for the velocity potential $\phi(x,y,t)$ is

$$\phi_{tt} = \phi_{xx} + \phi_{yy} \quad (y < 0),$$

$$\phi_y = 0 \ (y = 0, x < 0), \quad \phi_y = -1 \ (y = 0, 0 < x < a(t)),$$
$$\phi = 0 \ (y = 0, x > a(t)), \quad \phi = \phi_t = 0 \ (y < 0, t = 0).$$

The function $a(t)$ describes the position of the contact point, it is unknown in advance and has to be determined together with the velocity potential. In the non-dimensional variables the position of the entering body is given by the equation $y = M[f(x) - t]$, $x > 0$ where $f(x)$ describes the body shape and $f(0) = 0$, $(df/dx)(0) = 0$. For the problem considered a moment T_1 can be distinguished such that the pressure distribution over the contact region and the function $a(t)$ are given by indivisible analytical expressions when $t > T_1$. For example, $T_1 = 5$ for a parabolic shape ($f(x) = \frac{1}{2}x^2$), and $T_1 = 2$ for a box-like structure. The algebraic equations for $a(t)$ and for the corresponding function $a_i(t)$ in the Wagner theory have the forms

$$\int_0^a \frac{f(x)dx}{\sqrt{a-x}} = 2ta^{\frac{1}{2}} - \frac{2}{3}a^{\frac{3}{2}}, \qquad \int_0^{a_i} \frac{f(x)dx}{\sqrt{a_i - x}} = 2ta_i^{\frac{1}{2}}.$$

In the case of a parabolic shape

$$a(t) = \frac{1}{8}(\sqrt{240t + 25} - 5), \quad a_i(t) = \frac{1}{2}\sqrt{15t}.$$

It is worth noticing that for large times, $t \gg 1$, we obtain

$$a_i(t) - a(t) = \frac{5}{8} + O(t^{-\frac{1}{2}}).$$

The Wagner approach predicts that the pressure $p_i(x,0,t)$ over the contact spot is

$$p_i(x,0,t) = \frac{2}{\pi}\frac{a_i^{\frac{1}{2}}(t)}{(a_i(t) - x)^{\frac{1}{2}}}\frac{da_i}{dt}(t).$$

The expression for the acoustic pressure has a similar form where $a_i(t)$ must be replaced by $a(t_n(x,t))$, $t_n(x,t)$ is the solution of the equation $a(t_n) + t_n = x + t$. The hydrodynamic force

$F(t)$ calculated within the framework of the acoustic theory tends to the corresponding value $F_i(t)$ given by the Wagner approach

$$F(t) = F_i(t) + O(t^{-\frac{1}{2}}), \quad F_i(t) = \frac{15}{2\pi} \quad (t \to \infty).$$

These estimates indicate that the acoustic effects are long-term ones, and they may be distinquished mainly in the forms of pulsations amplitude of which decays slowly with time.

4. Extreme loads on an entering body

The pressure magnitude can be very high at the initial stage of the water impact. But for a blunt body those pressures are localized in space, the contact region where they are applied is therewith very small. That is why the total hydrodynamic force at the initial acoustic stage is of the same order of magnitude as that calculated within the framework of the incompressible liquid model (Wagner approach). Consider the case when the entering body is deformable and its position at the moment t is given by $y = y_b(x,t)$ in the dimensional variables. Here $y_b(0,0) = 0$, $y_b(x,0) > 0$ where $| x |> 0$, $y_{bx}(0,t) = 0$, $y_b(-x,t) = y_b(x,t)$, $y_{bt}(x,0) = -V$. The Wagner approach predicts that the hydrodynamic force on the entering body is given by

$$F(t) = -2aa_t\rho \int_0^{\frac{\pi}{2}} y_{bt}(a \sin\theta, t) d\theta - 2a^2\rho \int_0^{\frac{\pi}{2}} y_{btt}(a \sin\theta, t)\cos^2\theta d\theta \tag{1}$$

where ρ is the liquid density, $2a(t)$ is the dimension of the wetted part of the body, which is determined by the transcendental equation

$$\int_0^{\frac{\pi}{2}} y_b(a(t)\sin\theta, t)d\theta = 0. \tag{2}$$

For an undeformable body (1) leads to the well-known result $F(t) = \pi\rho a a_t V(t)$ where $V(t)$ is the body velocity. Differentiating (2) in time, one finds

$$a_t(t) \int_0^{\frac{\pi}{2}} y_{bx}(a(t)\sin\theta, t)\sin\theta d\theta + \int_0^{\frac{\pi}{2}} y_{bt}(a(t)\sin\theta, t)d\theta = 0. \tag{3}$$

Equations (1), (3) show that $F(t) \to \infty$ when the first integral in (3) tends to zero. The latter is impossible in the classical theory where $y_{bx}(x,t) > 0$ where $| x |> 0$. But this case is of possibility for special shapes of the body and for an elastic bottom as well.

We consider the entry of a rigid body where $y_b = f(x) - Vt$, $f(x) = \alpha x - \beta x^2 + \gamma x^3$, $0 < x < a_*$, $\alpha > 0$, $\beta > 0$, $\gamma > 0$, $f(-x) = f(x)$, a_* is such that the first integral in (3) is zero when $a(t_*) = a_*$. It was revealed that the Wagner theory can be applied to this shape of the entering body and $F(t) \to \infty$ when $t \to t_* - 0$ if $1 < 4\alpha\gamma/\beta^2 < \pi^2/8$. This case corresponds to the bow flare slamming when the inclination of the flare section is large.

High velocities $a_t(t)$ of the contact region expansion as a reason for high hydrodynamic force values were detected in the problem of wave impact onto an elastic plate. The calculations were carried out for a model where the lowest mode of the plate deflection was taken into account only. It is important to notice that the solution cannot be continued for $t > t_*$ where $a_t(t_*) = \infty$. The reason is that the pressures are very high at this stage not only near contact points but over whole contact region. Therefore, acoustic effects are expected to be of importance at this stage.

DISCUSSION

Tulin, M.: I seem to remember that the acoustic pressure on a blunt body at water entry is limited to ρuc. Is that consistent with your result for the parabolic blunt body?

Korobkin, A.: The present results for the parabolic contour show that the pressure at the top of the contour is equal to the "water hammer" pressure ρuc at the impact moment. The expression for the acoustic pressure, which is presented in the paper, is valid for $t > T_1$. The corresponding expression for the time interval $0 < t < T_1$ is more complicated and is not presented here.