# Interaction of a Large Three-dimensional Body with Waves and Currents by THOBEM

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## INTRODUCTION

Reliable predictions of wave forces and run-up on large offshore structures in combined waves and currents are of practical importance in the design and operation of these structures in the actual sea conditions. The same formulation can also be applied to the evaluation of forces and wave drift damping on a drifting body with small forward velocity. Recently, several numerical methods for this problem have been developed in the frequency domain (Emmerhoff & Sclavounos, 1992; Nossen et al, 1991; Teng & Eatock Taylor, 1994). The frequency domain analysis is in general mathematically complicated and cannot easily be extended to arbitrary lateral or bottom boundaries or fully nonlinear problem. This kind of problem can be more straightforwardly treated in the time-domain computer program. Isaacson & Cheung (1992) recently applied time domain analysis to solve wave-current-body interaction problem using constant panel method. When constant panels are used, the computation of the second-order spatial derivatives of the potential on the boundary surface is known to be inaccurate. In view of this, a time domain analysis based on higher order boundary element method (**THOBEM**) is developed.

In this paper, the effects of uniform steady currents or small forward velocity on wave-frequency and mean drift forces on and run-up around large three-dimensional structures are investigated. If the current or forward velocities are small, the effects of flow separation are unimportant and the problem can be formulated within the framework of potential theory. In our perturbation approach, both wave slope parameter $\varepsilon$ and current speed parameter $\delta$ are assumed to be small so that the steady wave system at $O(\varepsilon\delta^2)$ is insignificant compared to wave-current-body interaction terms at $O(\varepsilon\delta)$. The $O(\delta)$ problem is the so-called double body potential problem that satisfies zero-normal-velocity free surface condition. To solve $O(\delta)$ and $O(\varepsilon\delta)$ problems, Rankine sources are distributed over the entire boundary surface. The resulting boundary integral equation is integrated at each time step. The free surface boundary condition was updated using Adams-Bashforth method. At the intersection of the free surface and radiation boundaries, discontinuous elements are employed to circumvent singularity problems. At the open boundary, the Sommerfeld/Orlanski radiation condition (Orlanski, 1976) was used. For illustration, wave forces and run-up on a bottom-mounted vertical cylinder are presented.

## MATHEMATICAL FORMULATION

For analysis, Cartesian coordinate system with the origin on the mean free surface and z axis positive upward is used. The space fixed coordinates are denoted by $\vec{X}=(X,Y,Z)$, whereas the body fixed coordinates by $\vec{x}=(x,y,z)$. Then, the boundary conditions of the velocity potential $\Phi(X,Y,Z,t)$ satisfying the Laplace equation are given by

$$\frac{\partial \Phi}{\partial t}+\frac{1}{2}\nabla\Phi\cdot\nabla\Phi+g\zeta=0, \qquad \text{on} \quad y=\zeta \tag{1}$$

$$\frac{\partial \zeta}{\partial t}+\frac{\partial \Phi}{\partial x}\frac{\partial \zeta}{\partial x}+\frac{\partial \Phi}{\partial y}\frac{\partial \zeta}{\partial y}-\frac{\partial \Phi}{\partial z}=0, \qquad \text{on} \quad y=\zeta \tag{2}$$

$$\vec{n}\cdot\nabla\Phi=\vec{n}\cdot U \qquad \text{on the body surface} \tag{3}$$

where, $g$ is the gravitational acceleration, $U$ the current velocity (or forward speed), $\vec{n}$ the outward normal vector, and $\zeta$ the instantaneous free-surface elevation. If we define the velocity potential $\phi(x,y,z,t)$ and free-surface elevation $\eta(x,y,t)$ with respect to the body fixed coordinate system, the following Lorentz transformation can be used:

$$\frac{\partial \Phi}{\partial t}=\left[\frac{\partial}{\partial t}-U\frac{\partial}{\partial x}\right]\phi \qquad \text{or} \qquad \frac{\partial \zeta}{\partial t}=\left[\frac{\partial}{\partial t}-U\frac{\partial}{\partial x}\right]\eta \tag{4}$$

Using (4) and Taylor expansion of (1) and (2) with respect to $z$=0, and introducing two small parameters, $\varepsilon=H/L$ and $\delta=U/\sqrt{ga}$ ($H$=wave height, $L$=wavelength, $a$=radius), we can systematically expand the velocity potential as the following perturbation series:

$$\phi(\vec{x},t)=\underbrace{\varphi_b(\vec{x},t)}_{O(\delta)}+\underbrace{\varphi_1(\vec{x},t)}_{O(\varepsilon)}+\underbrace{\varphi_{11}(\vec{x},t)}_{O(\varepsilon\delta)}+\underbrace{\varphi_2(\vec{x},t)}_{O(\varepsilon^2)}+\ldots \tag{5}$$

$$\eta(\vec{x},t)=\underbrace{\eta_1(\vec{x},t)}_{O(\varepsilon)}+\underbrace{\eta_{11}(\vec{x},t)}_{O(\varepsilon\delta)}+\underbrace{\eta_2(\vec{x},t)}_{O(\varepsilon^2)}+\ldots \tag{6}$$

The subscripts *b*,*1* and *2* represent double-body, first-order and second-order potentials and wave elevations respectively, while the subscript *11* denotes the wave-current interaction terms. We assume that both $\varepsilon$ and $\delta$ are small, and will neglect those terms higher than $\varepsilon\delta^2$. The O($\delta$) and O($\varepsilon$) problems are well-known double body flow and first-order diffraction problems, respectively. The free-surface conditions at O($\varepsilon\delta$) are given by

$$\frac{\partial \varphi_{11}}{\partial z}-\frac{\partial \eta_{11}}{\partial t}=-U\frac{\partial \eta_1^s}{\partial x}+\frac{\partial \varphi_b}{\partial x}\frac{\partial \eta_1}{\partial x}+\frac{\partial \varphi_b}{\partial y}\frac{\partial \eta_1}{\partial y}-\eta_1\frac{\partial^2 \varphi_b}{\partial z^2} \tag{7}$$

$$\frac{\partial \varphi_{11}}{\partial t}+g\eta_{11}=U\frac{\partial \varphi_1^s}{\partial x}-\frac{\partial \varphi_b}{\partial x}\frac{\partial \varphi_1}{\partial x}-\frac{\partial \varphi_b}{\partial y}\frac{\partial \varphi_1}{\partial y} \tag{8}$$

where, $\varphi_1^s$ and $\eta_1^s$ are the first-order scattering potential and wave elevation.

## NUMERICAL METHODS

A direct boundary integral equation method based on the distribution of simple sources is developed for our numerical solution. Distributing basic singularities over the entire boundary surface, the method can be used for arbitrary bottom topography and lateral restrictions. The integral equation for the unknown potential is given by

$$C(p)\varphi(p)=\int_\Gamma\left[\frac{\partial G(p,q)}{\partial n}\varphi(q)-G(p,q)\frac{\partial \varphi(q)}{\partial n}\right]d\Gamma_q=0 \tag{9}$$

where,$G=1/R_1+1/R_2+1/R_3+1/R_4$, $R_1$ is basic source and $R_2,R_3,R_4$ are image sources with respect to respective symmetric planes, $C$ is solid angle, and $p$ and $q$ are field and source points. By employing quadratic isoparametric elements and shape functions, the discrete form of equation (9) was solved at each time step. To calculate the velocity potential and its normal derivatives on the free surface, Adams-Bashforth numerical integration scheme was employed. At the open boundary, Sommerfeld/Orlanski radiation boundary condition was used in the form; $\varphi_t+c\varphi_n=0$. This outgoing wave condition combined with the dynamic free-surface condition (8) was used to calculate the phase velocity $c$. This time dependent phase velocity calculation could be failed when $\varphi_t, \varphi_n \rightarrow 0$. To avoid this problem and to get a robust estimate of $c$, a numerical scanner (Jagannathan, 1988) was implemented.

With the solutions at each time step, wave forces on the body can be determined by carrying out an integration of the pressure over the instantaneous wetted body surface, which can further be perturbed about the still water level. Then, the first-order oscillatory force and second-order steady force are given by

$$F = -\rho \int_{S_b} (\frac{\partial \varphi}{\partial t} + \nabla \varphi_c \cdot \nabla \varphi) \cdot \vec{n}\, dS \tag{10}$$

$$\bar{F} = -\frac{1}{2}\rho \langle \int_{S_b} |\nabla \varphi|^2 \cdot \vec{n}\, dS \rangle + \frac{1}{2}\rho g \langle \int_{W_o} \eta^2 \cdot \vec{n}\, dW \rangle \tag{11}$$

where $\varphi$ and $\eta$ are total wave potential and wave elevation including incident waves, and $\varphi_c$ is double body potential including uniform flow. The symbol < > denotes a time average, and $S_b$, $W_o$ are mean body surface and its water line, respectively.

## RESULTS AND DISCUSSION

Numerical calculation was performed for a bottom-mounted vertical cylinder with radius $a$ for different wave numbers and current speeds. In order to avoid an abrupt initial condition and allow a gradual development of the flow field, a modulation function was applied to body boundary condition for the first two wave cycles. Figs. 2 show the time series of the first-order diffraction force and wave elevation on the lee side for $ka$=1. As can be seen in the figure, the time-integration scheme is very stable and the open boundary condition works well. The steady solution can be obtained after two or three wave cycles. Figs.3 show the wave run-up around the body for $ka$=1.0 and $\delta$=0.1,0.,-0.1. This result shows that current effects are significant to wave run-up. For Fig3(a), the velocity potentials, $\varphi_1$ and $\varphi_{11}$, are solved simultaneously, as suggested by Isaacson & Cheung (1992). Whereas, in Fig3(b), the first-order potential $\varphi_1$ was solved first, and then, the interaction potential $\varphi_{11}$ was subsequently solved with the computed first-order values in the right-hand side of (7),(8). Theoretically, the difference between the two is $O(\varepsilon\delta^2)$. The overall trend between the two looks similar, while there exists nontrivial difference in magnitude. Further numerical results including wave drift damping are on the way and will be presented later.

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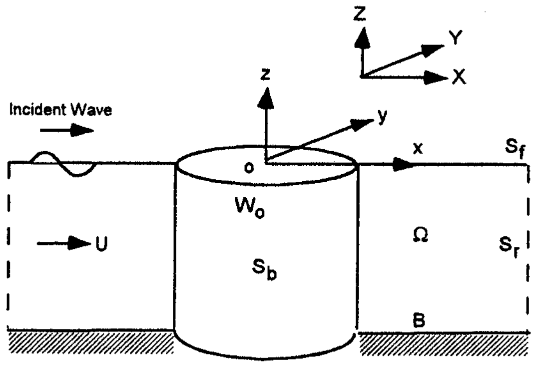
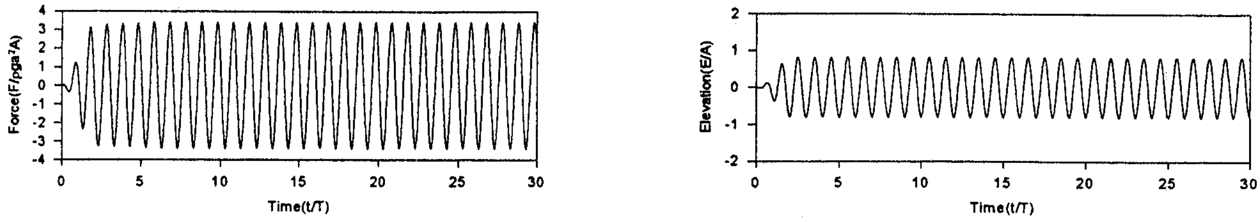
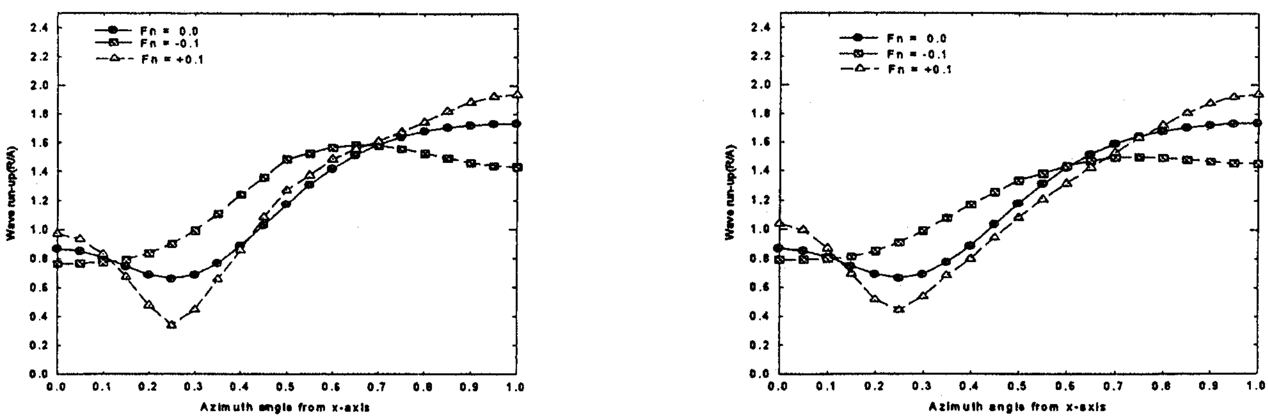



Fig. 1 Coordinate system




(a) (b)

Fig.2 First order (a)Force (b)Wave Elevation at θ=0.




(a) (b)

Fig.3 Wave run-up for ka=1.0