Application of a Higher Order Panel Method for Computing Higher Derivatives of the Steady Potential in a Ship Motions Program

by

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1 Introduction

Over the past several years Rankine Panel Methods have been applied to solve for the motions of a ship operating in waves and for the motions of structures interacting with waves and current. The papers of *Bertram (1990)*, *Nakos (1990)*, and *Zhao and Faltinsen (1989)* are representative of work done in this area. These methods decompose the total velocity potential into the steady potential due to the forward motion of the ship in calm water, the incoming wave potential, the diffraction potential due to the interaction of the motionless ship with the incoming waves and the radiation potentials due to the forced motions of the ship. Second derivatives of the steady potential appear in the expressions for the "m-terms," as defined in *Ogilvie and Tuck (1969)*, required for the kinematic boundary condition of the radiation problem. Higher order derivatives of the steady potential also appear in the pressure integrals needed to determine the forces and moments on the ship and in the expression for the boundary condition applied on the free water surface.

In most previous studies, first order panel methods, consisting of constant strength singularities on flat panels, are used to represent the ship hull and part of the free water surface. One of the difficulties associated with the use of first order panel methods is the inability of such an approach to predict second derivatives of the steady potential on the ship and water surfaces. *Bertram (1990)* avoids this problem by approximating all terms containing second derivatives of the stationary potential using a simple slender-body theory. *Zhao and Faltinsen (1989)* describe a couple possibilities for handling this problem, including computing the second derivative terms away from the body and using extrapolation to obtain the required values on the body surface. *Nakos (1990)*, who uses a bi-quadratic singularity distribution on a planar panel, avoids the computation of the second derivatives in the m-terms by developing and alternative expression through an application of Stoke's theorem. This work investigates the ability of a higher order panel method to predict the higher derivatives of the steady potential. Such a method should have the additional advantage of achieving comparable accuracy using fewer panels.

2 Description of the higher order panel method

The higher order panel method was first developed in two dimensions in order to determine the level of singularity strength and panel geometry representation required to accurately predict the second derivatives of the potential. During the 2-D investigation I examined the following alternatives for panel shape and for the source strength distribution on each panel:

- A flat panel with a constant source strength distribution (a first order method)
- A flat panel with a linear source strength distribution
- A parabolic panel with a constant source strength distribution
- A parabolic panel with a linear source strength distribution
- A parabolic panel with a parabolic source strength distribution

An ellipse with major axis length 1.0 and minor axis length of 0.5 was examined in uniform flow. Figure 1 shows a comparison of the second derivatives of the velocity potential, $\frac{\partial^2\phi}{\partial x^2}$ and $\frac{\partial^2\phi}{\partial x \partial y}$, from various panel methods for this flow. In this case 36 panels are used to model the ellipse. The velocity derivatives are computed at a panel control point on the ellipse 15% of the ellipse length behind the front end of the ellipse and along a line normal to the ellipse from that point. The geometry of the ellipse and the location of the points where the velocity derivatives were computed are shown in Figure 2. Figure 1 indicates that the improvement in the prediction of the second derivatives when a parabolic source distribution was used rather than a linear source distribution on a parabolic panel is small. This improvement was found to become even smaller as the number of panels was increased, which led to the decision to use a parabolic panel with a linear source strength to develop the 3-D method. It can also be seen from the figure that the first order panel method is unable to predict the second derivatives of the velocity distribution near the surface of the ellipse. Figure 3 shows the maximum error and the root mean square of the error in $\frac{\partial^2\phi}{\partial x^2}$ over the ellipse from the 2-D panel method using a parabolic panel shape and linear source strength distribution. This shows that the error in the second derivatives is $O(h)$ with this method and that an acceptable level of accuracy can be achieved with a reasonable number of panels. The results for the error in $\frac{\partial^2\phi}{\partial x \partial y}$ were almost identical.

The method was then extended to three dimensions. This method follows the approach described in *Hess (1979)*, with some improvements made to the way the panel geometry is defined. Each panel is represented by a parabolic surface. The equation describing the geometry of a panel in the local panel coordinate system is:

$$\zeta = A\xi + B\eta + C + P\xi^2 + 2Q\xi\eta + R\eta^2 \tag{1}$$

where the ξ-axis and η-axis lie in the plane tangent to the panel at the collocation point, which is near the panel centroid. The ζ-axis is normal to the panel at this point. The coefficients A, B, C, P, Q and R are determined by requiring the panel to pass exactly though its corner points while agreeing in a least squares since to the corner points of the neighboring panels. The source strength on each panel is represented by a bilinear distribution of the form:

$$\sigma(\xi,\eta) = \sigma_o + \sigma_x\xi + \sigma_y\eta \tag{2}$$

where σ_x and σ_y are the slopes of the source strength distribution in the ξ and η directions respectively.

The program was tested on the uniform flow past an ellipsoid. The ellipsoid had semiaxes of 1, 0.2 and 0.2 in the x, y and z directions respectively. The uniform onset flow was set parallel to the x-axis. Figure 4a shows the maximum error in the tangential velocity on the ellipsoid from a higher order and first order panel method using various numbers of panels. The root mean square of the error in the tangential velocity was also examined and showed similar results. For the higher order method the error in the tangential velocity is $O(h^2)$, and for the lower order method the error in the tangential velocity is $O(h)$. Since panels now cover a three-dimensional surface, $O(h^2)$, where h is the panel width, is equivalent to $O(1/N)$, where N is the total number of panels used. Similarly $O(h)$ is equivalent to $O(1/\sqrt{N})$. Better results were obtained with the higher order panel method when a panel grid with many quadrilateral panels at both ends was used as opposed to a more typical panel grid with a cluster of triangular panels at both ends. The higher order panel method results presented in this paper were all computed using the type of grid shown in the inset of Fig. 4a. Figure 5 shows the distribution of $\frac{\partial^2\phi}{\partial x^2}$ and $\frac{\partial^2\phi}{\partial x \partial z}$ on the surface of the ellipsoid along the line forming the intersection of the ellipsoid and the x-z plane. The higher order panel method appears able to predict all of the second derivatives of the potential well for this case. Fig. 4b shows the maximum error in $\frac{\partial^2\phi}{\partial x^2}$ over an ellipsoid for a first order panel method and a higher order panel method. The $\frac{\partial^2\phi}{\partial x^2}$ term was found to be the most difficult second derivative to predict for this case. The other second derivatives were all found to be better predicted. The results for the root mean square of this error were similar. For the first order panel method, results are shown for two methods of evaluating the second derivatives of the potential: evaluating these terms directly on the body surface, and evaluating the terms at a few points more than a half panel length from the body along a line normal to the panel and using

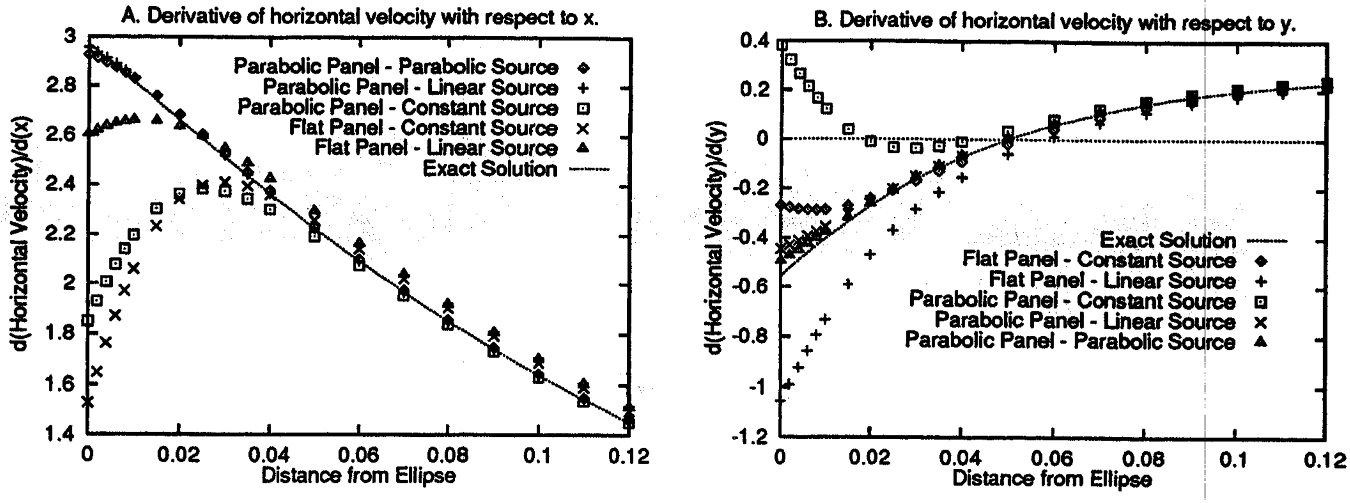

Figure 1: Error in $\frac{\partial^2\phi}{\partial x^2}$ over an ellipse from a first-order and higher-order panel method.

extrapolation to approximate the value on the body, as recommended in *Zhao and Faltinsen (1989)*. The higher order panel method is shown to achieve $O(h^{3/2})$ accuracy for the second derivatives of the potential. For the first order panel method the second derivative of the potential can only be approximated by computing the values away from the body and extrapolating to compute the value on the body surface. The error from this procedure does not decrease consistently with decreasing panel size, although it appears to approach $O(h)$ when a large number of panels is used. If the second derivatives are computed directly on the body using the first order method, the accuracy is at best $O(1)$ and actually appears to increase as the panel size decreases. From Fig 5a it can be seen that $\frac{\partial^2\phi}{\partial x^2}$ ranges from about -28 to 2 on the ellipsoid, so a maximum error of 1, corresponds to about 4%.

3 Conclusions and Work in Progress

A higher order panel method is shown to give accurate predictions of the second derivatives of the velocity potential for an ellipsoid in uniform flow. Work is in progress to incorporate this procedure into the framework of a program to determine ship motions in waves following the approach of *Bertram (1990)*. It is hoped that by the time of the Workshop results from this procedure can be shown for several ship flows, with comparisons made to results from the current method.

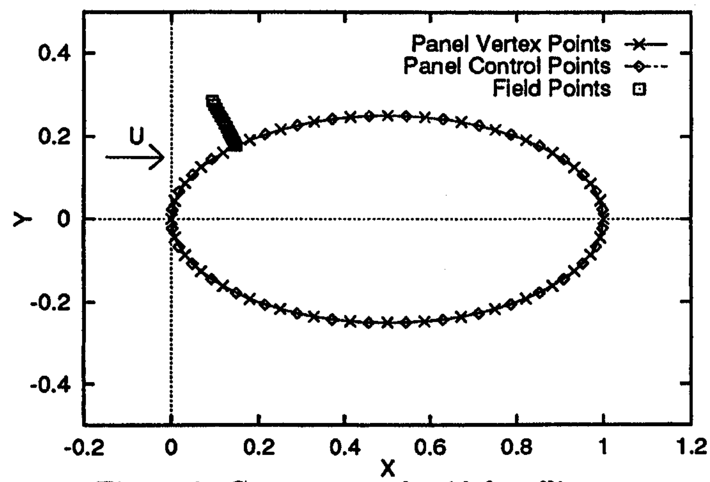

Figure 2: Geometry and grid for ellipse.

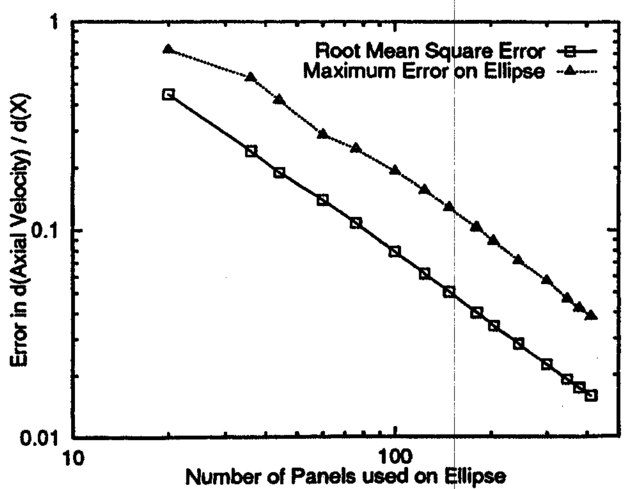

Figure 3: Maximum and RMS error in $\frac{\partial^2\phi}{\partial x^2}$ over ellipse.

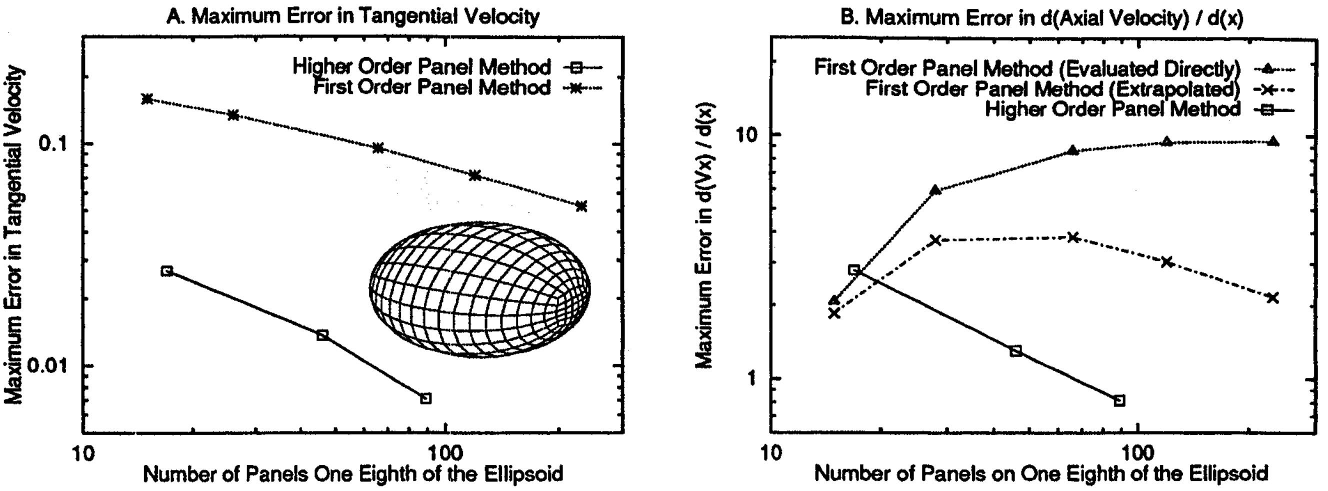

Figure 4: Maximum error in the tangential velocity and $\frac{\partial^2\phi}{\partial x^2}$ on an ellipsoid in uniform flow computed using various panel method schemes.

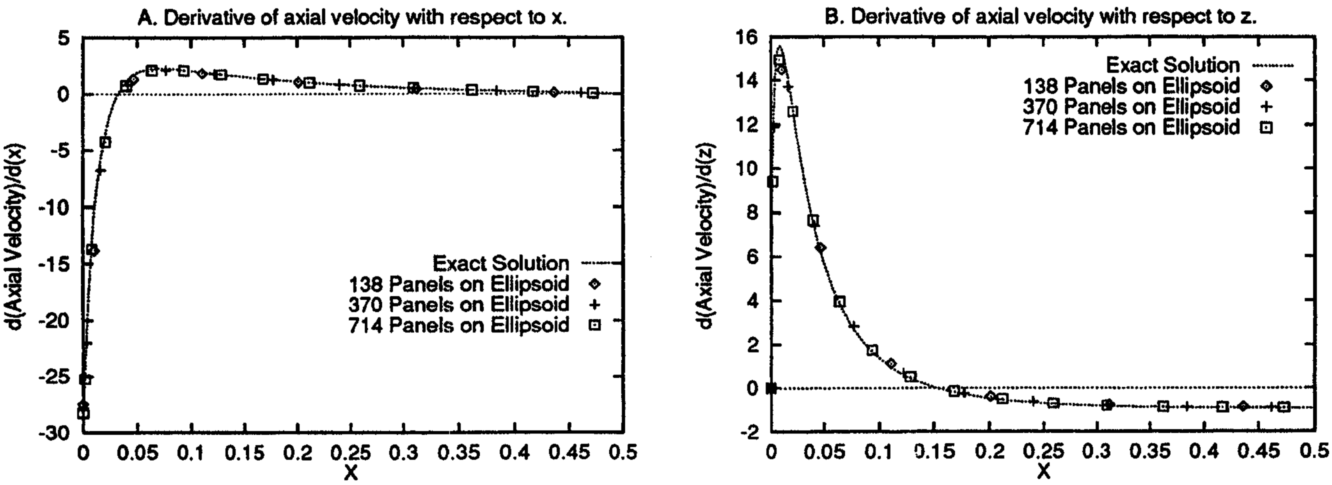

Figure 5: Distribution of $\frac{\partial^2\phi}{\partial x^2}$ and $\frac{\partial^2\phi}{\partial x\partial z}$ over an ellipsoid computed from a higher order panel method.

References

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DISCUSSION

Yeung, R. W.: I agree with Hughes that it is appropriate to pursue the quest for a higher order method. Scott Coakley of my group has just completed an iso-parametric bi-cubic spline-based method for free surface flows. In some of our tests for flow about ellipsoids, we were able to achieve accuracy of one to two orders higher using only 64 or fewer "panels". The convergence with respect to panel size is also very much faster. Thus, I see there are definite merits of going in this direction. Coakley is in the process of writing a manuscript on his latest work.