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Nonlinear Shallow Water Flow on a Three-Dimensional Deck

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Water shipping on deck and the resulting water sloshing inside the deck well is dangerous to small vessels with a large open deck area. In case where a large amount of water is trapped in the deck well, the vessel's transverse stability is dramatically reduced and can lead to capsize. The present research is to study the nonlinear shallow water flow on a three-dimensional deck due to ship motions in six degrees of freedom. Related work has been conducted experimentally and numerically by Adee and Caglayan (1982) and again numerically by Pantazopoulos (1988) using the Random Choice method. The Random Choice method uses a random sampling technique, so that a phase error can be introduced to the hydraulic jump in the shallow water flow. It requires that the Courant number should be less than unity in order to keep the numerical scheme stable. However, if the Courant number is very small, the numerical scheme could give unstable solutions (Adee and Caglayan, 1982). The Flux Difference method was applied to compute the water flow on a 2-D deck (Huang and Hsiung, 1994). This method was developed based on the Flux-Vector Splitting method orginally given by Steger and Warming (1981) to compute shock waves in gas dynamics. Alcrudo et al. (1992) used this method to compute open channel flows. It is able to capture discontinuities (such as hydraulic jumps in open channel flow and deck flow) in the solutions. The present work is the extension of the authors' previous study on water flow on a 2-D deck.

A ship-fixed coordinate system $oxyz$ is used with coordinates attached on the deck bottom plane of rectangular shape, and the origin is at the centre of the bottom. The oz axis is vertical upward. The ship's rotational motions are represented by Euler's angles (e_1, e_2, e_3) of the ship in space. The instantaneous translational velocities of ship motion along the $ox-$, $oy-$ and $oz-$directions are u_1, u_2 and u_3, and the rotational velocities about axes parallel to ox, oy and oz and passing through the centre of gravity are u_4, u_5 and u_6, respectively. The velocity of the water particle is denoted by $\vec{v} = (u, v, w)$. The fluid motion is governed by the continuity equation and Euler's equations, subjected to the boundary conditions: $w = \frac{\partial\zeta}{\partial t} + u\frac{\partial\zeta}{\partial x} + v\frac{\partial\zeta}{\partial y}$ and $p = 0$ on the free surface $z = \zeta$; and $\vec{v}\cdot\vec{n} = 0$ on the deck's bottom and sides. Initially, the ship is assumed to be at the upright position and the fluid is motionless. If the water depth is sufficiently shallow compared with the horizontal dimension of the deck space, we can assume that (i) the deck bottom is flat and the sides of deck is perpendicular to the bottom; (ii) w is a small quantity compared with u and v; and (iii) $u = u(x, y, t)$ and $v = v(x, y, t)$. Then, the governing equations can be written as:

$$\frac{\partial\zeta}{\partial t} + \frac{\partial}{\partial x}(u\zeta) + \frac{\partial}{\partial y}(v\zeta) = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = q_0 + q_1\zeta + q_2\frac{\partial\zeta}{\partial x} + q_3\zeta\frac{\partial\zeta}{\partial x} \tag{2}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=r_0+r_1\zeta+r_2\frac{\partial\zeta}{\partial y}+r_3\zeta\frac{\partial\zeta}{\partial y} \tag{3}$$

where

$$q_0=-gsin(e_2)-\dot{u}_1+2u_6v+(u_5^2+u_6^2)(x-x_g)+(\dot{u}_6-u_4u_5)(y-y_g)+(u_4u_6-\dot{u}_5)z_g \tag{4}$$

$$q_2=-gcos(e_1)cos(e_2)-\dot{u}_3-2(u_4v-u_5u)-(u_4u_6\dot{u}_5)(x-x_g)-(u_5u_6-\dot{u}_4)(y-y_g)-(u_4^2+u_5^2)z_g \tag{5}$$

$$r_0=-gsin(e_1)sin(e_2)-\dot{u}_2-2u_6u-(u_4u_5+\dot{u}_6)(x-x_g)+(u_4^2+u_6^2)(y-y_g)+(u_5u_6-\dot{u}_4)z_g \tag{6}$$

$$q_1=-u_4u_6,\qquad q_3=u_4^2+u_5^2,\qquad r_1=-u_5u_6,\qquad r_2=q_2\quad\text{and}\quad r_3=q_3 \tag{7}$$

and $(x_g,\ y_g,\ z_g)$ is the centre of gravity of the ship.

Let $q=g\zeta$, the governing equation is expressed in terms of the flux vector as follows:

$$\frac{\partial\vec{W}}{\partial t}+\frac{\partial\vec{F}}{\partial x}+\frac{\partial\vec{G}}{\partial y}=[D]\frac{\partial\vec{W}}{\partial x}+[E]\frac{\partial\vec{W}}{\partial y}+\vec{f}_0+\vec{g}_0 \tag{8}$$

in which,

$$\vec{W}=\begin{pmatrix}q\\uq\\vq\end{pmatrix},\quad \vec{F}=\begin{pmatrix}uq\\u^2q+\frac{1}{2}q^2\\uvq\end{pmatrix},\quad \vec{G}=\begin{pmatrix}vq\\uvq\\v^2q+\frac{1}{2}q^2\end{pmatrix} \tag{9}$$

$$[D]=\begin{pmatrix}0&0&0\\0&qD_{22}&0\\0&0&0\end{pmatrix},\quad [E]=\begin{pmatrix}0&0&0\\0&0&0\\0&0&qE_{33}\end{pmatrix},\quad \vec{f}_0=\begin{pmatrix}0\\f_{02}\\0\end{pmatrix},\quad\text{and}\quad \vec{g}_0=\begin{pmatrix}0\\0\\g_{03}\end{pmatrix}, \tag{10}$$

with $D_{22}=1+q_2/g+qq_3/g^2$, $E_{33}=1+r_2/g+qr_3/g^2$, $f_{02}=qq_0+q^2q_1/g$, and $g_{03}=qr_0+q^2r_1/g$.

In numerical computation, the deck space is devided with $m\times n$ nodes and the governing equation is split into two equations:

$$\frac{\partial\vec{W}}{\partial t}+\frac{\partial\vec{F}}{\partial x}=[D]\frac{\partial\vec{W}}{\partial x}+\vec{f}_0 \tag{11}$$

and

$$\frac{\partial\vec{W}}{\partial t}+\frac{\partial\vec{G}}{\partial y}=[E]\frac{\partial\vec{W}}{\partial y}+\vec{g}_0 \tag{12}$$

At each time step, instead of solving the 2-D governing equation for $m\times n$ nodes, we solve $m+n$ 1-D equations along the x- and y- directions separately using the Fractional Step Method (Marchuk, 1982). Equations are first solved along the x-direction; and it is assumed that $\vec{W}(x,y_j,t_n)=\vec{W}^n_{ij}$ and $\vec{F}(x,y_j,t_n)=\vec{F}^n_{ij}$ are constants for $x\in[x_i-\frac{1}{2}\Delta x,x_i+\frac{1}{2}\Delta x]$ and $y=y_j$ for $j=1,2,\cdots,n$. u', v' and q', are found, and $\vec{W}(x,y,t)$ and $\vec{G}(x,y,t)$ can be calculated at nodal points using u', v' and q'. Then we solve the equations along the y-direction by assuming that $\vec{W}(x_i,y,t)=\vec{W}'_{ij}$ and $\vec{G}(x_i,y,t)=\vec{G}'_{ij}$ are constants for $y\in[y_j-\frac{1}{2}\Delta y,y_j+\frac{1}{2}\Delta y]$ and $x=x_i$ for $i=1,2,\cdots,m$. Similar to the work given by Huang and Hsiung (1994) for the 2-D deck space, the finite difference equations can be derived as follows:

$$\vec{W}'_{i,j}=\vec{W}^n_{i,j}-\beta_1\Delta\vec{F}^*_{i-\frac{1}{2},j}+\beta_1\Delta\vec{F}^*_{i+\frac{1}{2},j},\qquad\text{for}\qquad j=1,2,\cdots,n \tag{13}$$

$$\vec{W}^{n+1}_{i,j}=\vec{W}'_{i,j}-\beta_2\Delta\vec{G}^*_{i,j-\frac{1}{2}}+\beta_2\Delta\vec{G}^*_{i,j+\frac{1}{2}},\qquad\text{for}\qquad i=1,2,\cdots,m \tag{14}$$

where $\beta_1=\frac{\Delta x}{\Delta t}$ and $\beta_2=\frac{\Delta y}{\Delta t}$. The Flux-Difference Splitting method together with the Superbee limiter are applied to $\Delta\vec{F}$ and $\Delta\vec{G}$ to obtain a stable finite difference scheme. The expressions for the flux difference are too lengthy to be included in this abstract. Details will be presented at the Workshop.

The source code has been developed, and was validated against the published experimental data by Adee and Caglayan (1982) and the exact solution of bore propagation by Stoker (1948). Effect of the Superbee limiter was also investigated. It was applied to the wave motion resulting from the impulse boundary movement in the shallow water. A single wave was developed and propagated in the direction of the boundary movement. The single wave has all the properties of a soliton in the Boussinesq theory except that as the wave form propagates, its front becomes steep and its tail relatively flat. This is because the nonlinearity instead of the dispersive effect is dominant in the governing equations.

The shallow water flow in a deck well of 1.0 m. by 1.0 m. is calculated for the mean water depth 0.06 m. The deck flow is excited by roll motion. The numerical results shows that the water surface is a "solid" horizontal surface at a rolling frequency much less than the primary resonant frequency. As the roll frequency is close to the first resonant frequency, a single bore appears. Between the first and second resonant frequencies, the water surface is like a solid surface parallel to the deck bottom. When the rolling frequency is close to the second resonant frequency, standing waves are formed with the wave length equal to the deck width. As the rolling amplitude increases, the wave amplitude is increased and small short waves appears on top of standing waves. If the rolling frequency is further increased to about the third resonant frequency, a big solitary wave travels back and forth in the deck well. We also calculated the water flow on deck caused by a slow amplitude-modulated roll motion: $e_1 = a_1(1+a_2 sin(\Omega t))sin(\omega t)$, where Ω is the modulation frequency and a_2 is the modulation ratio. It is interesting to note that at the rolling frequency ω near the second resonant frequency, $\Omega/\omega = 0.2, a_1 = 3$ deg. and $a_2 = 1.2$, the time history of the wave elevation shows a motion of the period $2/\Omega$, see Fig. 1. However, if Ω/ω is slightly increased to 0.204 and $a_1 = 4$ deg., the wave motion is no longer periodical, as shown in Fig. 2. Whether this is due to the flow instability is still under investigation.

The computation is also carried out for the mean water depth 0.1 m. The deck is forced to oscillate in roll and pitch motions with both amplitudes 5 deg. Fig. 3 shows the wave profile at $t = 3.0$ sec. and frequency 7.0 rad/sec. The wave profile is shown in Fig. 3 and the velocity distribution is in Fig. 4 at $t = 3.0$ sec. When the frequency $\omega = 4.0$ rad/sec. is near the primary resonant frequency of the deck flow, two perpendicular bores appear in the deck space and there also exists a oblique bore caused by the interaction of the two perpendicular bores (Fig. 5). The bore in the greater water depth travels faster than that in the smaller water depth. Deck flow under the heave excitation is given in Fig. 6 at $t = 2.5$ sec., where the excitation frequency equals the second resonant frequency of the deck flow. At the beginning of computation, small disturbances in the x- and y-directions are applied simultaneously to the deck. The disturbance has the amplitude of 0.005 m. and only lasts for 0.05 sec. The surface wave caused by the disturbance does not decay and moves in the x-direction. In the y-direction, the wave profile looks like a solid surface oscillating about the x-axis. In other frequencies, the wave profile is always as a solid horizontal plane even though a disturbance is applied.

References

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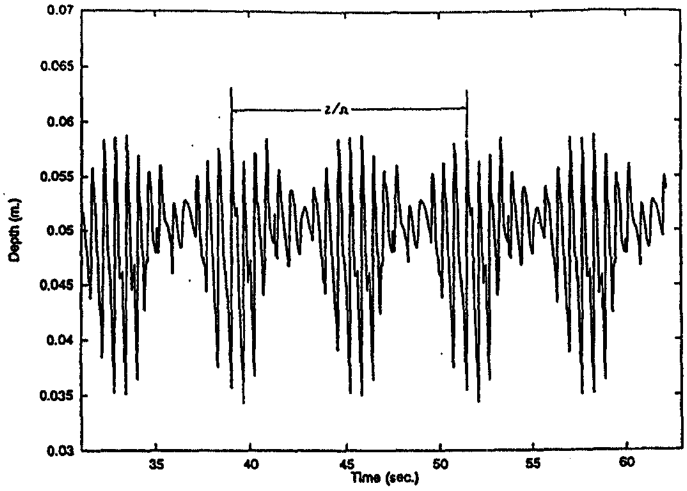

Fig.1 Wave motion under roll excitation

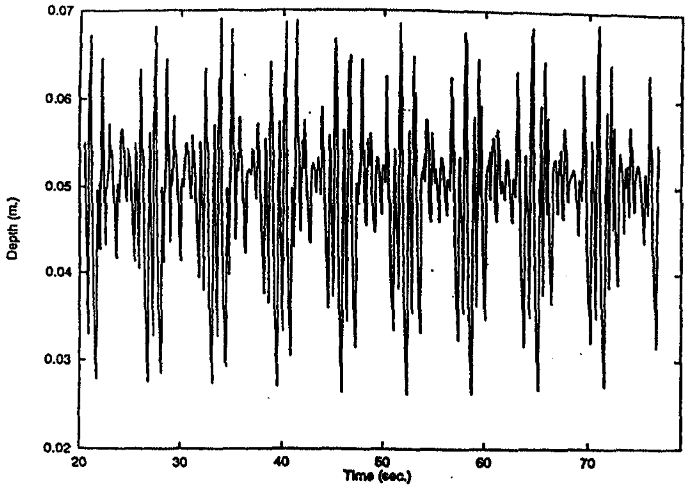

Fig.2 The aperiodic wave motion

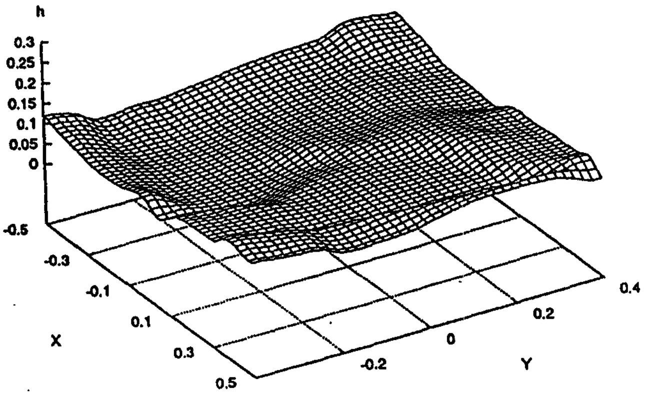

Fig.3 Wave profile at $t = 3.0$ sec. $\omega = 7.0$ rad/sec.

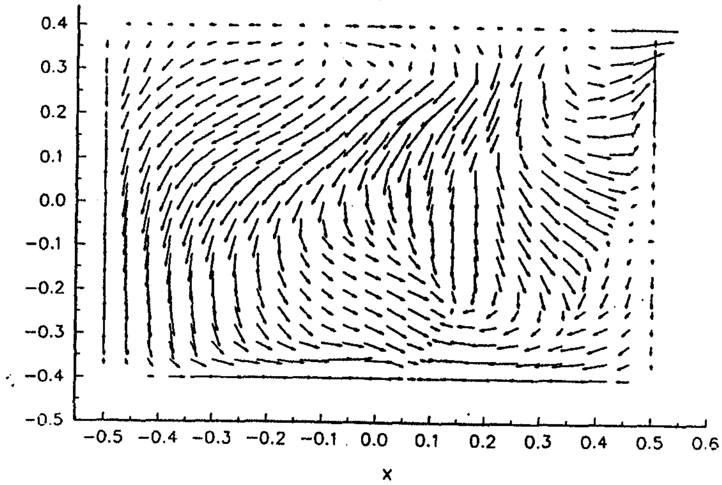

Fig.4 Velocity distribution at $t = 3.0$ sec.

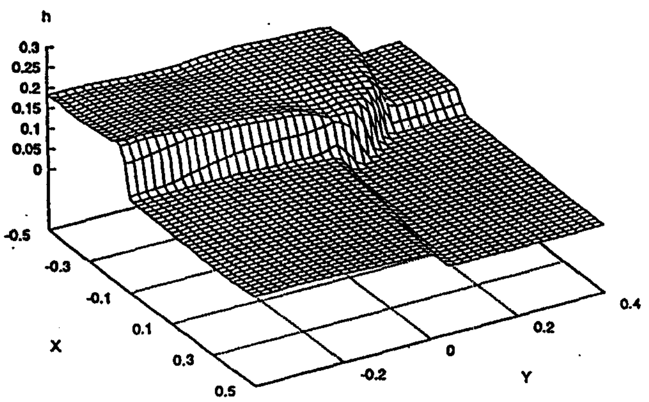

Fig.5 Wave profile at $t = 4.0$ sec. $\omega = 4.0$ rad/sec.

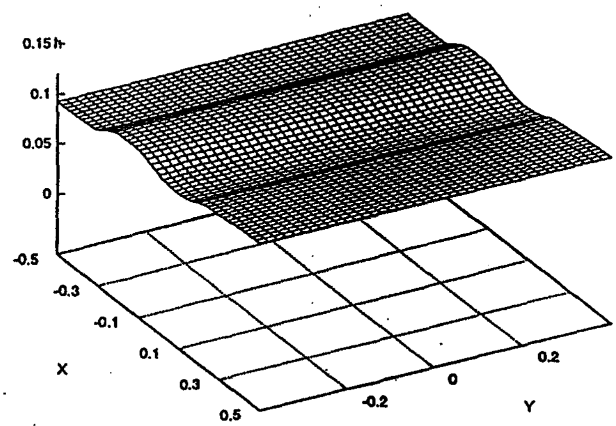

Fig.6 Wave profile due to heave, $t = 2.5$ sec. $\omega = 7.8$ rad

DISCUSSION

Korobkin, A. : The present method can be applied after simple modifications to the problem of liquid drop impact onto a rigid plate (plane case). The latter problem was the subject of many numerical studies. But common numerical schemes have a great artificial viscosity. That is why the comparison of the present approach with results from the drop impact problem can be helpful to understand more clearly the peculiarities of the method suggested by the authors.

Huang, Z. J. & Hsiung, C. C.: Thank you for your interesting and useful comments.

Yeung, R. W.: The governing equations that you are solving are non-dissipative and an energy conservation theorem can be derived. Did you apply an energy check to your solution? If not, I would think this would be a worthwhile effort.

Huang, Z. J. & Hsiung, C. C.: We did not apply an energy check to the solution. However, in order to avoid unphysical stationary jumps (those in which energy increases across the jump) a flux limiting function, the Superbee limiter, is introduced in our computational scheme. The Superbee limiter is also able to eliminate oscillations within the solutions, and thus reduces smearing of the computed bore.

King, A. C.: How do you advance the surface height in time on the boundary of the domain?

Huang, Z. J. & Hsiung, C. C.: This is a time-domain solution with 500 time-steps. The water surface is discretized with a grid of 50 x 40 elements. The initial conditions are: 1) the ship is at the upright position; and 2) the water is calm. With a prescribed ship motion as the external excitation, at each time-step, the free surface configuration is computed, including the intersection with the boundary (deck wall) as well as the wave elevation.