Analysis of the forces and the responses of floating bodies with a slow yaw-motion

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We consider wave drift damping of floating bodies such as oil-platforms and ships. Earlier works have considered the wave forces and the drift damping which is acting on floating bodies with a slow translatory motion, see e.g. Zhao and Faltinsen (1989), Nossen, Grue and Palm (1991), Emmerhoff and Sclavounos (1992). The aim of the present contribution is to study wave forces and wave drift damping acting on floating bodies with a slow rotation about the vertical axis. The present theory accounts for the linear responses of the floating body, which is a generalization of the contributions by Newman (1993) and Grue and Palm (1994), who considered the diffraction problem only.

We consider the fluid flow and the linear body responses in the relative frame of reference rotating with the slow angular velocity of the body. The Coriolis force is accounted for. The slow angular velocity of the body is assumed to be much smaller than the wave frequency of the incoming waves. The magnitude of the rotation angle may be arbitrary, however. The first step is to find the linear fluid motion, the linear wave forces and the linear body responses. Next we consider the wave drift damping due to the slow rotation, which is the ultimate goal of the analysis.

Viscous effects are neglected, and the fluid is assumed incompressible and irrotational, potential theory may then be applied. Furthermore we assume that the amplitude of the incoming waves and the body motions are small compared to the size of the body. Green's theorem is adopted to formulate the integral equations for the diffraction and radiation problems. Applying a perturbation method, the problem is solved to leading order in the slow angular velocity. The numerical calculation is performed by a 3-D panel method, which is applicable to bodies of arbitrary shape.

1 The integral equations

The fluid velocity $\mathbf{v}$ is described in a frame of reference (x, y, z) rotating with the angular velocity $\mathbf{\Omega} = \Omega\mathbf{k}$, with the z-axis being vertical upwards. We may then write

$$\mathbf{v} = \nabla\Phi + \nabla\psi^{(2)} + \Omega\mathbf{w} \tag{1}$$

Here, $\mathbf{w} = \nabla\chi_6 - \mathbf{k} \times \boldsymbol{x}$ represents the fluid motion when there are no waves, and $\psi^{(2)}$ is a time-average second order potential being proportional to the wave amplitude squared. Φ is the linear potential proportional to the wave amplitude, and is written

$$\Phi = Re\{\frac{Aig}{\omega}\phi_D e^{i\omega t} + \frac{d}{dt}(\xi_j e^{i\omega t})\phi_j\} \tag{2}$$

where A and ω are the amplitude and frequency of the incoming waves, respectively, and ϕ_D is the diffraction potential. ξ_j, $j = 1, \ldots, 6$ are the amplitude of motion in the jth mode, and ϕ_j are the corresponding radiation potential. The assumption that the body is slowly rotating means that $\Omega/\omega \ll 1$. Thus, the problem has two time scales: a fast time scale proportional to $1/\omega$, and a slow time scale, proportional to $1/\Omega$. We now define $\epsilon \equiv \omega\Omega/g$, and introduce the following perturbation expansions

$$\xi_j = \xi_j^0 + \epsilon\xi_j^1 \tag{3}$$

$$\phi_j = \phi_j^0 + \epsilon\phi_j^1 \tag{4}$$

$$\phi_D = \phi_I + \phi_7^0 + \epsilon\phi_7^1 \tag{5}$$

In what follows we will keep terms proportional to ϵ, and neglect terms proportional to $\Omega^2, \dot{\Omega}$. In (5), $\phi_I = e^{Kz - iKR\cos(\beta-\theta)}$ represents the incoming waves. Here $K = \omega^2/g$ is the wavenumber, R and θ are polar coordinates ($x = R\cos\theta, y = R\sin\theta$), and $\beta(t)$ is the angle between the propagation direction of the incoming waves and the x-axis in the rotating frame of reference. Now $\frac{d\beta}{dt} = -\Omega$ and $\frac{d}{dt}(\xi_j e^{i\omega t}) = (i\omega\xi_j - \Omega\xi_{j,\beta})e^{i\omega t}$, the free surface condition for ϕ_j then reads
ϵ^0:

$$-K\phi_j^0 + \phi_{j,z}^0 = 0 \quad \text{at} \quad z = 0 \tag{6}$$

ϵ^1:

$$\begin{aligned} -K\xi_j^0\phi_j^1 + \xi_j^0\phi_{j,z}^1 - 2i\xi_{j,\beta}^0\phi_j^0 - 2i\xi_j^0\phi_{j,\theta}^0 \\ +2i\xi_j^0\nabla_h\chi_6 \cdot \nabla_h\phi_j^0 + i\xi_j^0\nabla_h^2\chi_6\phi_j^0 = 0 \quad \text{at} \quad z = 0 \end{aligned} \tag{7}$$

Here ∇_h denotes the horisontal gradient. The body boundary condition is
ϵ^0:

$$\phi_{j,n}^0 = n_j \quad \text{on } S_B \tag{8}$$

ϵ^1:

$$\phi_{j,n}^1 = \frac{1}{iK}m_j \quad \text{on } S_B \tag{9}$$

where (n_1, n_2, n_3) denote the Cartesian components of the normal vector $\mathbf{n}$, pointing out of the fluid domain, $(n_4, n_5, n_6) = \boldsymbol{x} \times \mathbf{n}$, $(m_1, m_2, m_3) = -((\mathbf{n} \cdot \nabla)\mathbf{w} + 2\mathbf{k} \times \mathbf{n})$ and $(m_4, m_5, m_6) = -((\mathbf{n} \cdot \nabla)(\boldsymbol{x} \times \mathbf{w}) + 2\boldsymbol{x} \times (\mathbf{k} \times \mathbf{n}))$. Far away from the body ϕ_j must behave as outgoing waves. The diffraction problem is formulated by Grue and Palm (1994). Here we formulate the radiation problem. To solve the boundary value problem, it is convenient to introduce ϕ_j^{11}, ϕ_j^{12} and

ϕ_j^{13} so that $\xi_j^0\phi_j^1 = \xi_{j,\beta}^0\phi_j^{11} + \xi_j^0(\phi_j^{12} + \phi_j^{13})$. The potentials ϕ_j^{11}, ϕ_j^{12} are particular solutions of the B.V.P., and are expressed in terms of ϕ_j^0, namely $\phi_j^{11} = 2i\phi_{j,K}^0$, $\phi_j^{12} = 2i\phi_{j,K\theta}^0$. We may then show that the set of integral equations for $\psi_j^1 = \phi_j^{12} + \phi_j^{13}$ are

$$\left\{ \begin{array}{c} 4\pi\psi_j^1 \\ 2\pi\psi_j^1 \end{array} \right\} + \int_{S_B} \psi_j^1 G_{,n}^0 dS = -\int_{S_B} (G^1 n_j + \mathbf{w}\cdot\nabla G^0 \frac{n_j}{iK} - \phi_j^0 G_{,n}^1) dS$$
$$- \int_{S_F} i\phi_j^0 (2\nabla_h \chi_6 \cdot \nabla_h G^0 + G^0 \nabla_h^2 \chi_6) dS \qquad (10)$$

where the Green function $G = G^0 + \epsilon G^1$, and $G^1 = 2iG_{,K\theta}^0$, and S_B, S_F denote the wetted body surface and the free surface, respectively.

2 The wave forces

By integrating the pressure, the components F_i of the linear hydrodynamic force $\mathbf{F}$ may be written

$$F_i = Re\{(A(X_i^0 + \epsilon(\frac{i}{K}X_{i,\beta}^0 + X_i^1))$$
$$+ Kf_{ij}^0\xi_j^0 + \epsilon(Kf_{ij}^0\xi_j^1 + Kf_{ij}^1\xi_j^0 + 2i(Kf_{ij}^0)_{,K}\xi_{j,\beta}^0))e^{i\omega t}\} \qquad (11)$$

X_i^0 are the components of the exiting force and f_{ij}^0 represent the added mass and damping coefficients, when $\Omega = 0$. Furthermore

$$X_i^1 = \rho g \int_{S_B} (\phi_7^1 - \frac{i}{K}\mathbf{w}\cdot\nabla(\phi_I + \phi_7^0)) n_i dS \qquad (12)$$

and

$$f_{ij}^1 = \rho g \int_{S_B} (\psi_j^1 - \frac{i}{K}\mathbf{w}\cdot\nabla\phi_j^0) n_i dS \qquad (13)$$

It may be shown that f_{ij}^1 satisfy the relation

$$f_{ij}^1 = -f_{ji}^1 \qquad (14)$$

which is a generalization of the Timman-Newman relations. An alternative to (12) is to express X_i^1 by the ϕ_i^{13}-part of the radiation potential by

$$X_i^1 = \rho g \int_{S_\infty} (\phi_I \phi_{i,n}^{13} - \phi_i^{13}\phi_{I,n}) dS \qquad (15)$$

which is generalized far-field Haskind relations. S_∞ is the vertical cylinder with radius increasing to infinity. As in Grue and Palm (1994) the second order time-average yaw moment may be expressed as

$$\overline{M_z} = \epsilon\rho\frac{g}{\omega}\frac{\partial}{\partial\beta}\overline{\int_V \mathbf{k}\cdot(\boldsymbol{x}\times\mathbf{v}')dV} - \rho\int_{S_\infty} \overline{v_{,\theta}' v_{,n}'} R dS \qquad (16)$$

being proportional to the square of the amplitude. Here $\mathbf{v}' = \mathbf{v} + \Omega\mathbf{k}\times\boldsymbol{x}$, and V is the fluid volume. This expression may be further developed to find the wave drift damping moment.

3 The body responses

Finally the equation of motion for ξ_i^1 in the rotating frame of reference takes the form

$$(-K(gM_{ij} + f_{ij}^0) + c_{ij})\xi_j^1 = A(X_i^1 + \frac{i}{K}X_{i,\beta}^0)$$
$$+(Kf_{ij}^1 - 2igM_{ij}^c)\xi_j^0 + 2i(gM_{ij} + (Kf_{ij}^0)_{,K})\xi_{j,\beta}^0 \qquad (17)$$

where M_{ij} is the body inertia matrix, and c_{ij} is the matrix of hydrostatic coefficients. The term involving M_{ij}^c is the Coriolis term, and $M_{ij}^c = -M_{i+1,j}$ when $j = 1,4$, $M_{ij}^c = M_{i-1,j}$ when $j = 2,5$ and $M_{3j}^c = M_{6j}^c = 0$

To verify the model, the body response ξ_1^1 and ξ_3^1 due to a half immersed sphere moving in incoming waves such that the center of the sphere describe a circle in the horizontal plane, are computed when $\beta = \pi$, and then compared to results obtained from the translatory problem corresponding to our example. With 400 panels on the body the relative difference is about 2 % when $0.25 \leq Ka \leq 1$, and a is the radius of the sphere.

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DISCUSSION

Zou, Z. J.: Higher order derivatives of velocity potential appear in the boundary conditions and integral equations. Did you calculate these terms directly or indirectly? Could you please give some details about this calculation?

Finne, S. & Grue, J.: The integrals where the *m*-terms in the body boundary condition for ϕ_j^1 appear are rewritten by using a variant of Stokes' theorem. Thus the body integral in the integral equation for ψ_j^1 only involves first order derivatives of the velocity potential χ_6. The term $\nabla_h^2 \chi_6 = -\chi_{6,zz}$ in the free surface integral is computed by numerical difference. We here exploit that $\chi_{6,z} = 0$ at $z = 0$ leading to a robust numerical evaluation.