Ringing loads on gravity based structures

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Ringing is of concern in survival conditions for gravity based structures (GBS) and tension leg platforms (TLP) in deep water. Ringing is caused by extreme waves exciting transient resonance response of structural modes. The relevant resonance periods are significantly lower than the peak period of the wave spectrum. The interesting natural period for a GBS is about one third of T_p. Basic studies on ringing loads on a fixed vertical and infinitely long circular cylinder in deep water incident waves were reported by Faltinsen, Newman and Vinje (1994) (FNV) and by Newman (1994) (N). FNV assumed regular incident waves and N considered irregular waves. Their procedure will be generalized to a monotower with non-circular cross-sections varying along the cylinder axis.

Cartesian coordinates (x,y,z) are defined with $z = 0$ in the mean water level. Positive z is upwards. The x-y planes and y-z planes are symmetry planes for the cross-section of the monotower. The surface normal vector $\vec{n} = (n_1, n_2, n_3)$ is positive into the fluid domain. Incident longcrested irregular waves propagating along the x-axis are studied. The characteristic wave amplitude A and structural cross-dimension a are $O(\varepsilon)$, where $\varepsilon \ll 1$. The characteristic wave length is $O(1)$. The cylinder (monotower) is slender and fixed. The cross-sectional shape can vary slowly along the cylinder length so that $n_3 = O(\varepsilon)$. Potential flow is assumed. The total velocity potential is written as $\phi = \phi_I + \phi_S + \psi$, where ϕ_I is the incident wave potential. ϕ_S can be found by slenderbody theory and matched asymptotic expansions. We can write $\phi_D = \phi_I + \phi_S$ as

$$\phi_D = \phi_{I0} + u(x + \phi_{11}) + u_x(0.5x^2 + \phi_{21}) + w\phi_{25} + f(z,t) \tag{1}$$

in the near field of the cylinder. Here ϕ_{I0},u,u_x,w are functions of z and time t and the values of ϕ_I,$\partial\phi_I/\partial x$, $\partial^2\phi_I/\partial x^2$, $\partial\phi_I/\partial z$ at $x = 0$,$y = 0$. ϕ_{11},ϕ_{21} and ϕ_{25} satisfy a 2-D Laplace equation in the cross-sectional plane and the body boundary conditions.

$$\frac{\partial\phi_{11}}{\partial N} = -n_1, \qquad \frac{\partial\phi_{21}}{\partial N} = -xn_1, \qquad \frac{\partial\phi_{25}}{\partial N} = -n_3 \tag{2}$$

Here $\vec{N} = (n_1, n_2)$. ϕ_{11} has a 2-D dipole behaviour far away from the cylinder and matches with a far-field 3-D horizontal dipole distribution along the cylinder axis. ϕ_{25} and part of ϕ_{21} have a far-field sourcelike behaviour. $f(z,t)$ is a consequence of matching with a far-field 3-D source distribution along the cylinder axis. It follows from the boundary value problem that $\phi_{11} = O(\varepsilon)$, $\phi_{21} = O(\varepsilon^2)$, $\phi_{25} = O(\varepsilon^2)$, $f(z,t) = O(\varepsilon^3)$.

ψ is a consequence of that ϕ_s does not satisfy the free surface condition to correct order of magnitude. The variation of ψ along the cylinder length is the same order of magnitude as the variation in x and y. So ψ satisfies a 3-D Laplace equation. Due to the strong z-variations of ψ it is essential that the formulation of the free surface condition for ψ is based on perturbations about the linear incident free surface elevations and not about $z = 0$. The free surface condition is

$$g\frac{\partial\psi}{\partial z} = -2uu_t\left(2\frac{\partial\phi_{11}}{\partial x} + (\nabla\phi_{11})^2\right) - 0.5u^3\left(2\frac{\partial^2\phi_{11}}{\partial x^2} + 2\frac{\partial}{\partial x}(\nabla\phi_{11})^2 + \nabla\phi_{11}\cdot\nabla(\nabla\phi_{11})^2\right) \tag{3}$$

on $z = \zeta_{I1}$.

Here g is the acceleration of gravity, ζ_{I1} is the linear incident free surface elevation at $x = 0, y = 0$. The body boundary condition is $\partial\psi/\partial n = 0$. ψ will be asymptotically small when $(z - \zeta_{I1}) = o(\varepsilon)$. It follows from (3) that $\psi = O(\varepsilon^3)$.

The horizontal loads per unit length due to ϕ_D only can be written as

$$F' = \rho A(z)\frac{Du}{Dt} + a_{11}(z)\frac{\partial u}{\partial t} + w\frac{\partial}{\partial z}(ua_{11}(z)) + O(\varepsilon^5) \tag{4}$$

for a totally wetted cross-section. Here $A(z)$ =cross-sectional area, ρ = mass density of the fluid, D/Dt =substantial derivative and $a_{11} = \int_\Sigma \phi_{11}n_1 ds$ is the two-dimensional added mass in surge. $\sum$ is the cross-sectional surface curve. (4) can be found by starting out with Bernoullis equation for the pressure. The first term is the Froude-Kriloff force and follows by using the divergence theorem on the volume inside the body. The second term follows from the $-\rho\partial\phi_s/\partial t-$ term and by noting that ϕ_{21}, ϕ_{25} and$f(z,t)$ will not contribute to the force. The last term can be derived by first noting that $0.5\int_\Sigma dsn_1(\nabla\phi_s)^2 = \int_\Sigma \frac{\partial\phi_s}{\partial n}\frac{\partial\phi_s}{\partial x}ds$. This follows from the divergence theorem. The right hand side of the last expression can be rewritten by the body boundary condition. We then combine the pressure forces resulting in the last term of (4) by first considering the force on a segment of length dz. This means we study $\rho w\int\int[(u\phi_{11})_z n_1 - (u\phi_{11})_x n_3]\,ds$. The last term in (4) follows now from Stokes theorem. The integration of the total pressure force which acts on the cylinder in the x-direction, can be decomposed into integrations from $z = -\infty$ to $z = 0$, from $z = 0$ to $z = \zeta_{I1}$ and from $z = \zeta_{I1}$ to $z = \zeta_{I1} + \zeta_2$. $\zeta_{I1} + \zeta_2$ is the local wave elevation at the cylinder surface correct to $O(\varepsilon^2)$. It includes both the effect of the incident waves and the locally scattered free surface. The contribution by integrating (4) from $z = 0$ to $z = \zeta_{I1}$ is

$$F'\zeta_{I1} + 0.5\zeta_{I1}^2\frac{\partial^2 u}{\partial t\partial z}(\rho A + a_{11}) + O(\varepsilon^6) \tag{5}$$

The vertical pressure gradient from $z = \zeta_{I1}$ to $z = \zeta_{I1} + \zeta_2$ is approximately hydrostatic. The resulting horizontal force correct to $O(\varepsilon^5)$ is

$$F_{HS} = -0.5\rho g\int_{\Sigma_1} n_1\zeta_2^2 ds = \rho u_t\int_{\Sigma_1} dsn_1(x + \phi_{11})\left[\zeta_{I2} - (u^2/g)\left(0.5(\nabla\phi_{11})^2 + \partial\phi_{11}/\partial x\right)\right] \tag{6}$$

where Σ_1 is Σ at $z = \zeta_{I1}$. ζ_{I2} is the second order part of the incident wave elevation at $x = 0, y = 0$. The horizontal force due to ψ can be written as

$$F^{(\psi)} = \rho\int\int_{S_B}(\psi_t + \nabla\phi_D\cdot\nabla\psi)n_1 dS + O(\varepsilon^6) \tag{7}$$

The body surface S_B extends from $z = -\infty$ to $z = \zeta_{I1}$. (7) can be rewritten by Green's second identity. We introduce ϕ_{11} as an auxiliary function and rewrite $\int\int_{S_B} \psi_t n_1 ds$ as $\int\int_{S_F} \phi_{11}\psi_{tz} ds$. Here S_F is the horizontal plane outside the cross-section at $z = \zeta_{I1}$. By using (4) and symmetry and antisymmetry properties of ϕ_{11} and its derivatives, it follows that

$$\rho \int\int_{S_B} \psi_t n_1 dS = -(3/g)u^2 u_t \rho \int\int_{S_F} dS\phi_{11}\left[\frac{\partial^2\phi_{11}}{\partial x^2} + \frac{\partial}{\partial x}\left((\nabla\phi_{11})^2\right) + 0.5\nabla\phi_{11}\cdot\nabla\left((\nabla\phi_{11})^2\right)\right] \tag{8}$$

By partial integration it follows that $\int\int \nabla\phi_D \cdot \nabla\psi n_1 dS = -u \int\int \psi \frac{\partial}{\partial s}\left(\frac{\partial}{\partial s}(x+\phi_{11})\right) dS$

The integration is over S_B and $\partial/\partial s$ is the tangential derivative along the body surface in the cross-sectional plane. We introduce an auxiliary potential ϕ_a that satisfies 2-D Laplace equation in x and y and the body boundary condition

$$\frac{\partial\phi_a}{\partial N} = \frac{\partial}{\partial s}\left(n_1\frac{\partial}{\partial s}(x+\phi_{11})\right) \tag{9}$$

The solution for a circular cylinder is $\phi_a = (a/r)^2\cos 2\theta$ where a is the cylinder radius and (r,θ) are polar coordinates so that $x = r\cos\theta$, $y = r\sin\theta$.

By using Green's second identity, the free surface condition, symmetry and antisymmetry properties of the integrand, it follows that

$$\rho \int\int_{S_B} \nabla\phi_D \cdot \nabla\psi n_1 dS = -(2/g)u^2 u_t \rho \int\int_{S_F} dS\phi_a\left(2\partial\phi_{11}/\partial x + (\nabla\phi_{11})^2\right) \tag{10}$$

$F^{(\psi)}$ can be interpreted as a point load moving with $z = \zeta_{I1}(t)$. Only 2-D potentials are needed in the calculations. Part of the S_F–integration in (8) and (10) can be made analytically by expressing ϕ_{11} as $\Sigma_{n=1}^{\infty} A_n r^{-(2n+1)}\cos(2n+1)\theta$ and ϕ_a as $\Sigma_{n=1}^{\infty} B_n r^{-2n}\cos 2n\theta$ outside a circle that encloses the body. The presented theory includes load terms of $O(A^3)$. The time dependence can be discussed similarly as in N. If we are interested in third harmonic load terms in deep water, it is sufficient to describe the incident wave held by a linear theory. If we want to find all the nonlinear load components described by the present theory, a third order theory for the irregular incident wave field is needed. The theory can be generalized to other wave headings, cross-sections without symmetry planes and a multicolumn GBS. The effect of body motion can be included. Generalization of the method for a TLP needs further studies. The effects of junctions between columns and pontoons, heave forces and roll and pitch motions should be evaluated. This can be done by matched asymptotic expansions. Let us as a simplified example consider a slender vertical circular cylinder with finite draft. 3-D flow corrections are needed at the lower end of the cylinder. One needs to solve boundary value problems with a semi-infinite cylinder to represent the inner-flow region near the bottom end. The resulting horizontal point load correctly to $O(\varepsilon^5)$ includes terms proportional to u_t, u_{tz}, uu_x and uw. This implies that the third order harmonic horizontal force terms are due to free surface effects. Similarly can be concluded from an analysis of the flow at the junctions between columns and pontoons. At least fourth order harmonic load terms are needed in the ringing analysis of a TLP. It should be noted that the present theory gives fourth order harmonic terms in roll and pitch moments about an axis close to the mean free surface.

This work started as a part of the Joint Industry Prospect "Higher order wave loads effects on large volume structures" and has been continued and further developed in a close cooperation with Professor J.N. Newman.

REFERENCES

Faltinsen, O.M., Newman, J.N., and Vinje, T.: "Nonlinear wave loads on a slender vertical cylinder", Jl. of Fluid Mechanics, in press , 1994.

Newman, J.N.: "Nonlinear Scattering of Long Waves by a Vertical Cylinder", Symposium in honor of Professor Enok Palm, Oslo, 17 Nov. 1994.

DISCUSSION

Grue, J.: In some experiments performed at our department, and in reports I have seen about generation of ringing responses, higher harmonic loads (and responses) are very small when the wave amplitude is below a certain value, but pronounced when the amplitude is above this value. Can you comment on this in relation to your theory?

Faltinsen, O. M: The ringing loads are proportional to wave amplitude cubed so they have a strong variation with the wave amplitude. There is no threshold value for the wave amplitude so that ringing loads do not exist below that amplitude. I believe this is consistent with experimental data that I have seen.

Rainey, R. C. T.: The perturbation scheme in F.N.V. produces, at successive orders, the same 1st, 2nd and (probably*) 3rd order wave loads, as does Stokes expansion, applied to a slender cylinder. So despite the assumption that "A/a = 0(1)", stressed in F.N.V., it appears that the wave loads could have been produced equally well using Stokes expansion (i.e. assuming $A/a \rightarrow 0$). Do you agree?

* See my abstract; also Malenica and Molin's numerical results, recently submitted to J.F.M.

Faltinsen, O.M.: The assumption A/a=0(1) implies that the free surface conditions are found by a perturbation of the dynamic and kinematic free surface conditions about a horizontal plane following the incident waves at the cylinder axis. Part of the solution varies strongly over a depth of 0(A) from that horizontal plane. I cannot see how one can get the same solution by assuming A/a is small and satisfying the free surface conditions in the conventional way on the mean free surface.

Tulin, M.: If the relevant parameters are g, d (diameter), a (wave amplitude) and σ (wave frequency), then the non-dimensional force must depend on two non-dimensional parameters: ($a\sigma^2/g$) and a/d. An equivalent set is: ak, kd. In your theory, the non-dimensional co-efficient seemed to depend on only one of these, kd. This suggests that the wave steepness, ak, is not a governing parameter (for instance, a certain value of ak is not required for the onset of ringing). Is this reasonably in accord with the facts?

Faltinsen, O. M. : It is true that a certain value of ak is not required for the onset of ringing. This is also in accordance with experimental facts that I have seen. However one would in practice define ringing to be a problem when the response is beyond a certain value.