Radiation and Diffraction Waves at High Froude Number

Makoto Ohkusu
Research Institute for Applied Mechanics, Kyushu University

It is possible to measure radiation and diffraction wave field generated by a ship advancing in waves, though they are invisible at tank test because of the existence of other waves such as steady waves and incident waves. Generally wave elevation is readily measured by simple and reliable instrumentation. Therefore the accuracy of the measured radiation and diffractin waves has less uncertainties than other quantities such as pressure and flow velocity. The comparison of the measured and the predicted radiation and diffraction wavefields will be more reliable test of hydrodynamic theories to predict the seakeeping of the ship.

Instanteneaous wave distribution around the ship does not provide a complete information of the radiation and diffraction waves. We need to know the distribution of the amplitude and phase of the wave motions. My method to obtain this information experimentally is to place several wave probes fixed to the water tank and on a line parallel to the track of the ship model with appropriate spacing between them. In this set up the probes come to the same location relatively to the ship model on different moments. In other words the probes record the wave elevation on several different moments at every location on the line paralell to the ship model track. From those records we derive the amplitude and the phase of the wave motion there.

By placing the probes on another line and repeating the experiments we construct the radiation and diffraction wave field $\eta(x,y)e^{i\omega t}$(as shown in Fig.1). Away from the ship they are given by

$$\begin{aligned}\eta =& \left[\int_{-\frac{\pi}{2}}^{\varphi-\frac{\pi}{2}} - \int_{\frac{\pi}{2}}^{\varphi+\frac{\pi}{2}}\right] F_1(\theta)e^{-ik_1(x\cos\theta+y\sin\theta)}d\theta \\ &+\left[\int_{\varphi-\frac{\pi}{2}}^{-\alpha_0} + \int_{\alpha_0}^{\varphi+\frac{\pi}{2}}\right] F_2(\theta)e^{-ik_2(x\cos\theta+y\sin\theta)}d\theta\end{aligned} \tag{1}$$

where

$$k_{1,2} = \frac{K_0}{2\cos^2\theta}(1-2\tau\cos\theta \pm \sqrt{1-4\tau\cos\theta}) \tag{2}$$

$$K_0 = \frac{g}{U^2} \qquad \varphi = \tan^{-1}\frac{y}{x} \qquad \alpha_0 = \cos^{-1}\frac{1}{4\tau} \qquad \tau = \frac{U\omega}{g} \tag{3}$$

The Fourier transform of $\eta(x,y)$ with respect to x is

$$\int_{-\infty}^{\infty}\eta(x;y)e^{\lambda x}dx = \begin{cases} 2\pi\dfrac{F_1(\theta)}{g_1(\theta)}e^{ik_1y\cdot\mathrm{sgn}(\cos\theta)\sin\theta} \\ \qquad \text{for } \lambda \ge K_0\tau \quad \text{or} \quad \lambda \le -\dfrac{K_0}{2}(1+2\tau+\sqrt{1+4\tau}) \\ \qquad \alpha_0 \le \theta \le \pi : \text{the solution of } \lambda = k_1\cos\theta \\ 2\pi\dfrac{F_2(\theta)}{g_2(\theta)}e^{-ik_2y\sin\theta} \\ \qquad \text{for } -\dfrac{K_0}{2}(1+2\tau-\sqrt{1+4\tau}) \le \lambda \le K_0\tau \\ \qquad \alpha_0 \le \theta \le \pi : \text{the solution of } \lambda = k_2\cos\theta \end{cases} \tag{4}$$

where

$$g_{1,2}(\theta) = \pm k_{1,2} \sin\theta / \sqrt{1 - 4\tau\cos\theta} \tag{5}$$

This transform at different wave number λ provides the value of either $F_1(\theta)$ or $F_2(\theta)$. In other words F_1 and F_2 have no common wave number in the x direction.

At higher Froude number diverging part of radiation and diffraction waves becomes dominant. The diverging waves propagate into the direction θ close to $\pi/2$ and their wave number in the x direction λ is very small. Naturally $F_{1,2}$ of the divergeng waves derived by the transform (4)(the longi-cut method) is less accurate because the information of η is limited for several ship lenghs of x. At higher speed of the ship the y-wise transform (the trans-cut) is plausible. F_1 and F_2, however, have the components with identical y-direction wave number μ at generally two different θ and θ^*. It follows that

$$\int_{-\infty}^{\infty} \eta(x,y) e^{i\mu y} dy = \begin{cases} 2\pi \dfrac{F_1(\theta)}{f_1(\theta)} e^{-ik_1(\theta)x\cos\theta} + 2\pi \dfrac{F_1(\theta^*)}{f_1(\theta^*)} e^{-ik_1(\theta^*)x\cos\theta^*} \\ \qquad \text{for } \mu > 4K_0\tau^2 \sin\alpha_0, \\ \qquad \alpha_0 \le \theta, \theta^* \le \pi : \text{two solutions of } \mu = k_1 \sin\theta \\ -2\pi \dfrac{F_2(\theta)}{f_2(\theta)} e^{-ik_2(\theta)x\cos\theta} + 2\pi \dfrac{F_1(\theta^*)}{f_1(\theta^*)} e^{-ik_1(\theta^*)x\cos\theta^*} \\ \qquad \text{for } 0 < \mu < 4K_0\tau^2 \sin\alpha_0, \\ \qquad \alpha \le \theta \le \pi : \text{the solution of } \mu = k_2 \sin\theta \\ \qquad \dfrac{\pi}{2} \le \theta^* \le \pi : \text{the solution of } \mu = k_1 \sin\theta \end{cases} \tag{6}$$

where

$$f_{1,2}(\theta) = \frac{k_{1,2}}{\cos\theta} \left(1 \pm \frac{\sin^2\theta}{\sqrt{1 - 4\tau\cos\theta}} \right) \tag{7}$$

The trans-cut transformation was investigated first in Naito and Zhang (1990). Their result does not include the terms depending on θ^* on the right hand side of (6). In order to obtain $F_1(\theta)$ and $F_2(\theta)$ separately we need to repeat the transformation (6) of the measured $\eta(x,y)$ at more than two different xs, say, $x = x_1, x_2, \cdots, x_M$.

Then $F_1(\theta^*)$ and $F_2(\theta)$ are determined by solving the following equations.

$$\begin{cases} \displaystyle\sum_{p=1}^{M} \int_{-\infty}^{\infty} \eta(x_p, y) e^{i\mu y} dy \cdot e^{ik_2(\theta)x_p\cos\theta} \\ \displaystyle\quad = -2\pi \frac{F_2(\theta)}{f_2(\theta)} M \\ \displaystyle\qquad + 2\pi \frac{F_1(\theta^*)}{f_1(\theta^*)} \sum_{p=1}^{M} e^{-ix_p(k_1(\theta^*)\cos\theta^* - k_2(\theta)\cos\theta)} \\ \\ \displaystyle\sum_{p=1}^{M} \int_{-\infty}^{\infty} \eta(x_p, y) e^{i\mu y} dy \cdot e^{ik_1(\theta^*)x_p\cos\theta^*} \\ \displaystyle\quad = -2\pi \frac{F_2(\theta)}{f_2(\theta)} \sum_{p=1}^{M} e^{-ix_p(k_2(\theta)\cos\theta - k_1(\theta^*)\cos\theta^*)} \\ \displaystyle\qquad + 2\pi \frac{F_1(\theta^*)}{f_1(\theta^*)} M \end{cases} \tag{8}$$

Similar equations are given for determing $F_1(\theta)$ and $F_1(\theta^*)$.

Normalized $F_2(\theta)$ obtained by two methods, the longi-cut and the trans-cut, is shown in Fig.2 for the radiation waves of heave mode. This is with a hull form slender and having the transom stern. Agreement of the results by both methods is almost perfect, suggesting the linear expression (1) of the wave field is valid. However the same fuctions for the diffraction waves do not agree as shown in Fig.3 except for θ less than 100°. F_2 by the longi-cut is not supposed to have good accuracy at θ close to 180° because wave data is extrapolated beyond some distance behind the ship model, while the x-wise length of the data used for the trans-cut analysis may not be enough long to give the accurate F_2 there. So this discrepancy must be studied further.

Fig.4 is for the diffraction waves at lower forward speed with non-slender hull form. Agreement of the values by two methods looks better, except for $\theta \sim 180°$.

It may be concluded from the results shown here that $F_2(\theta)$ is accurately derived by the longi-cut as well as the trans-cut methods for the wave components propagating forward ($\theta \leq 90°$) and the former is plausible because the measurement of waves is much easier.

References

1)Chapmans, R.B.(1976). Free surface effects for yawed surface-pirecing plate. J.Ship Research 20.

2)Daoud, N.(1975). Potential flow near to a fine ship's bow, Rep. No.177, Dep. Nav. Archt. Marine. Eng., Univ. of Michigan.

3)Naito,S. and Zhang,J.(1990), Research on unsteady wave field-Wave pattern analysis with transverse cut method-, J.the Kansai Society of Naval Architects, Japan No.213 (1990)

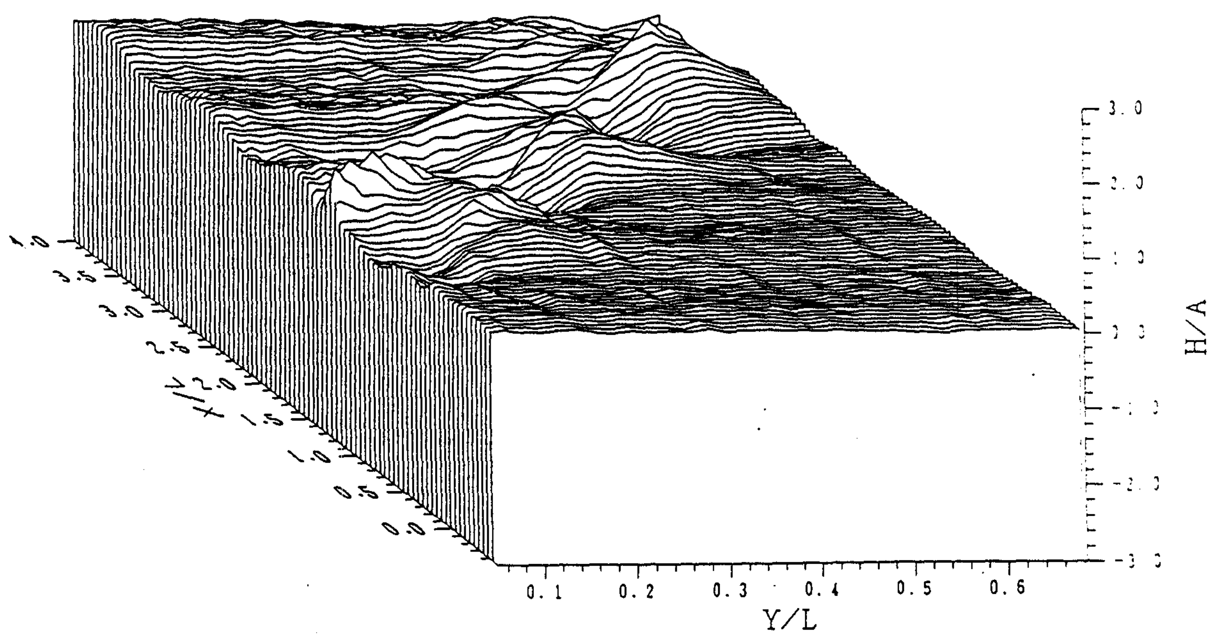

Fig.1 Diffraction Waves $\mathrm{Re}[\eta](F_N = 0.5, \lambda/L = 1)$

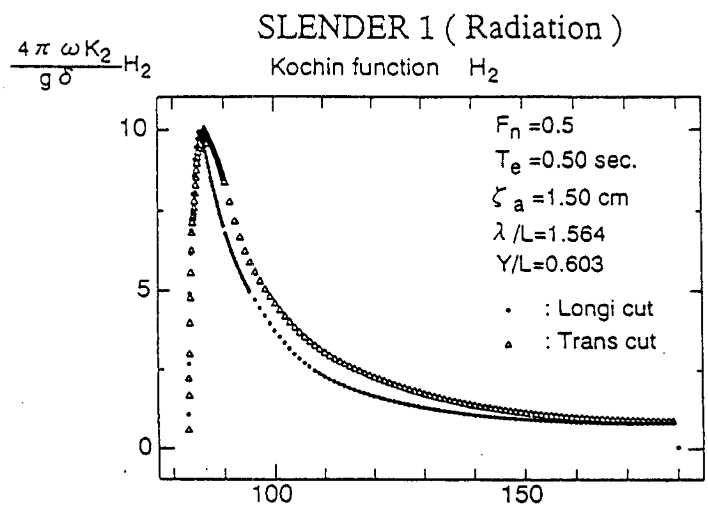

Fig.2 $F_2(\theta)$

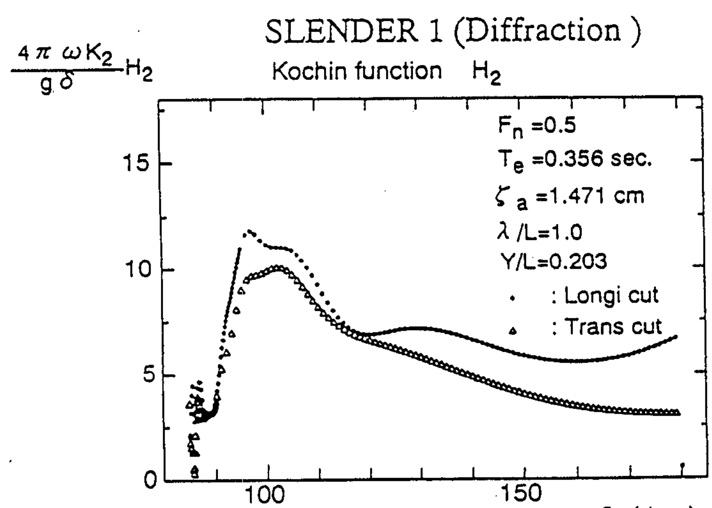

Fig.3 $F_2(\theta)$

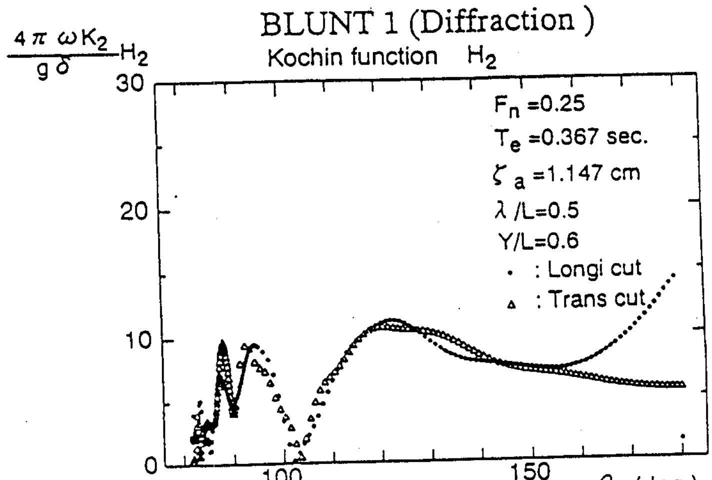

Fig.4 $F_2(\theta)$

DISCUSSION

JENSEN: I think in the towing tank there always will be energy dissipation due to wave breaking etc. Therefore it should be expected that wave analysis will always give smaller added resistance than direct force measurement.
OKHUSU: Yes, it is expected. But conjecture is not sufficient to describe such a big difference. Moreover experimental data suggest that some physical mechanism is working behind the phenomena. So we need a theory to tell how this happens.

EGGERS: The analysis underlying these pioneering investigations, I feel, has found little documentation in non-Japanese papers. Hence I like to amplify on it prior to putting questions.
A naive observer may wonder why the asymptotic formula (1) shows integration limits dependent on the polar angle ϕ (it is the proper limit for having the polar distance R tending to infinity) whereas (4) is obviously found from the limit of large y, which is adequate for longitudinal cut analysis. This may be seen from an integral representation of the complete velocity potential derived by Hanaoka, presented by Maruo [1], in a form of strong analogy to Michell's potential. Here the entire range for longitudinal wave number $k_1 \cos\theta$ (= speed waves) and $k_2 \cos\theta$ (=ring waves) as defined in my contribution to this workshop can be mapped in one-to-one correspondence on the axis: for the interval not covered (actually two intervals for τ small) Hanaoka's single integral terms show exponential rather than oscillatory dependence on y. The ensuing expression for wave resistance can be derived from the integral of average flow of x-impulse through the vertical plane enclosing the wave cut using closed form z-integration after using Parseval's identity for the x-Fourier integrals. Hence the resistance expression is really equivalent to the flux of impulse through the vertical control surface; if it shows a much lower value than the averaged horizontal force measured on the model, we must seek for some leakage of energy in the fluid domain. In our workshop contribution we could show that a criterion for nonlinear resonance interaction can be met; if we can prove that this is related to a mesurable physical process, one might speculate about wave energy being transferred to a tertiary component through such interaction which escapes measurements under the instrumental technique applied here. What is the author's feeling about the resistance discrepancy observed?

Ref. [1]: Inui T. and Maruo, H.: Vol. 2 of 60th Anniversary Series Soc. Nav. Arch. Japan 1957, page 17. (Here m_1 et m_2 should be taken as $4\omega_0^2/k_0$ for τ exceeding 1/4.)