NEW TREATING OF THE FREE-SURFACE AND DOWNSTREAM BOUNDARY CONDITIONS IN NUMERICAL COMPUTATIONS OF FREE-SURFACE FLOWS

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Introduction

When the free-surface flows are numerically simulated either by the boundary element method (BEM) or by the finite difference method (FDM), the free-surface boundary condition and the treatment of the downstream boundary must be dealt with to preserve the flow characteristics. The efficiency of the computation is another important factor to be met.

Here some methods of treating these boundary conditions are attempted both for BEM and FDM; (a) The double mesh method, (b) the additional inclusion of the 3rd derivative of the Taylor expansion for the free-surface condition, (c) an alternative downstream boundary condition in terms of an analytical expression and (d) a numerical wave absorber. (a) and (c) are for BEM and (b) and (d) for FDM. The results are compared and discussions are made to evaluate their accuracy and efficiency.

Free-Surface Boundary Condition

(a) The mesh size for BEM can be larger than that used in FDM, which is an advantage over FDM. This is because all the variables have been integrated over the panel in BEM and its size can be valid only for the integrated variables. However, in most computations, a single pannel scheme is used both for the integrated variables and for the finite expression.

Xu, Mori & Shin (1989) applied the double-mesh method where two different grid schemes are used on the free-surface; one is a coarse panel used for the integrated variables and the other is a finer one used for the finite difference expression of the free-surface boundary conditions. The results are successful within a moderate computing time.

The method is applied also to the computation of the Navier-Stokes solver by FDM. Fig.1 shows the comparison of the wave patterns of the Wigley Model. Case-A is the results by the conventional method whose grid numbers are 120x45x15 in the oncoming flow, the lateral and the vertical directions respectively. Case-B is by the double mesh method where two grid systems of (200x50x15) and (100x25x15) are used. In Case-B, the former grid system is the computation of the free-surface elevation and the latter for the momentum equation. This is becasue finer mesh is necessary to resolve the free-surface elevation while even a relatively coarse mesh may work for the momentum computation. The results of Case-B by the double mesh method is much well resolved although the CPU time for one step is less; 13.2s for case-A and 11.1s for case-B by HITAC-680H.

(b) The MAC method calculates the free-surface elevation by dislocating the particle by the local velocity. Although the method is believed general, the wave does not propagates efficiently.

The other method is to calculate the elevation from the Euler-type expression of the kinematic free-surface boundary condition. The 3rd order upwind differencing scheme is commonly used which consists of a central differencing term and a diffusion term (4th order central differencing). Its four-downstream-point expression is

$$c\frac{\partial f}{\partial x} = \frac{c}{6h}(-2f_{i-3} + 9f_{i-2} - 18f_{i-1} + 11f_i) \tag{1}$$

where f_i means the values at x_i and c is the convective velocity and h is a constant. The 3rd derivatives of the Taylor expansion given by

$$\frac{1}{6h}c\alpha(\phi_{i-3} - 3\phi_{i-2} + 3\phi f_{i-1} - \phi_i) \quad (2)$$

contributes to accelerate the propagation of wavy nature, where α is a constant. Its addition to (1) is expected to make the computation more efficient.

Fig.2 shows the wave profiles calculated by the proposed finite differencing scheme:(a) and compared with those by the original MAC method:(b). The wave is generated by the pressure acting on the free-surface and the results of the linear theory by Salvesen & von Kerczek(1977) is also shown for comparison. The development of the wave by the present method is much accelerated than that by the MAC method. It should be noted that the result by $\Delta x = 0.04$ gives almost the same results as those by $\Delta x = 0.02$. Eventually the computing time is reduced less than half.

Downstream Boundary Condition

(c) In BEM, either ϕ or ϕ_n has to be provided on the downstream open boundary as a boundary condition, where ϕ is the velocity potential. An attempt is made in Xu & Mori (1990) to provide ϕ_n invoking for the analytic relation ;

$$\phi_n = \phi_x|_{z=0}e^{K_0 z}. \quad (3)$$

In (3), $\phi_x|_{z=0}$ is estimated from the value of ϕ on the free-surface where K_0 is the wave number. The use of ϕ_n instead of ϕ makes the problem much easier because of the character of the kernel function. An analytical expression contributes much in numerical computations.

(d) When the upstream finite differencing scheme is used in FDM, a simple zero- or linear extrapolation is commonly used on the downstream boundary. In some cases, no special treatment is made at all; e.g., Dawson's Rankine source method(1977). However, reflection from the downstream boundary is unavoidable to some extent.

Here a method to assume a numerical wave absorber is introduced as Hinatsu(1991) where the vertical velocity component is suppressed for a certain range. This may be equivalent to a wave absorber by porous horizontal plates. Fig.3 shows the wave elevation due to the pressure acting on the surface. The absorbing condition is imposed for the range between x=4.5 and 7.5. No significant reflection can be seen and the steady state is attained within less time-step.

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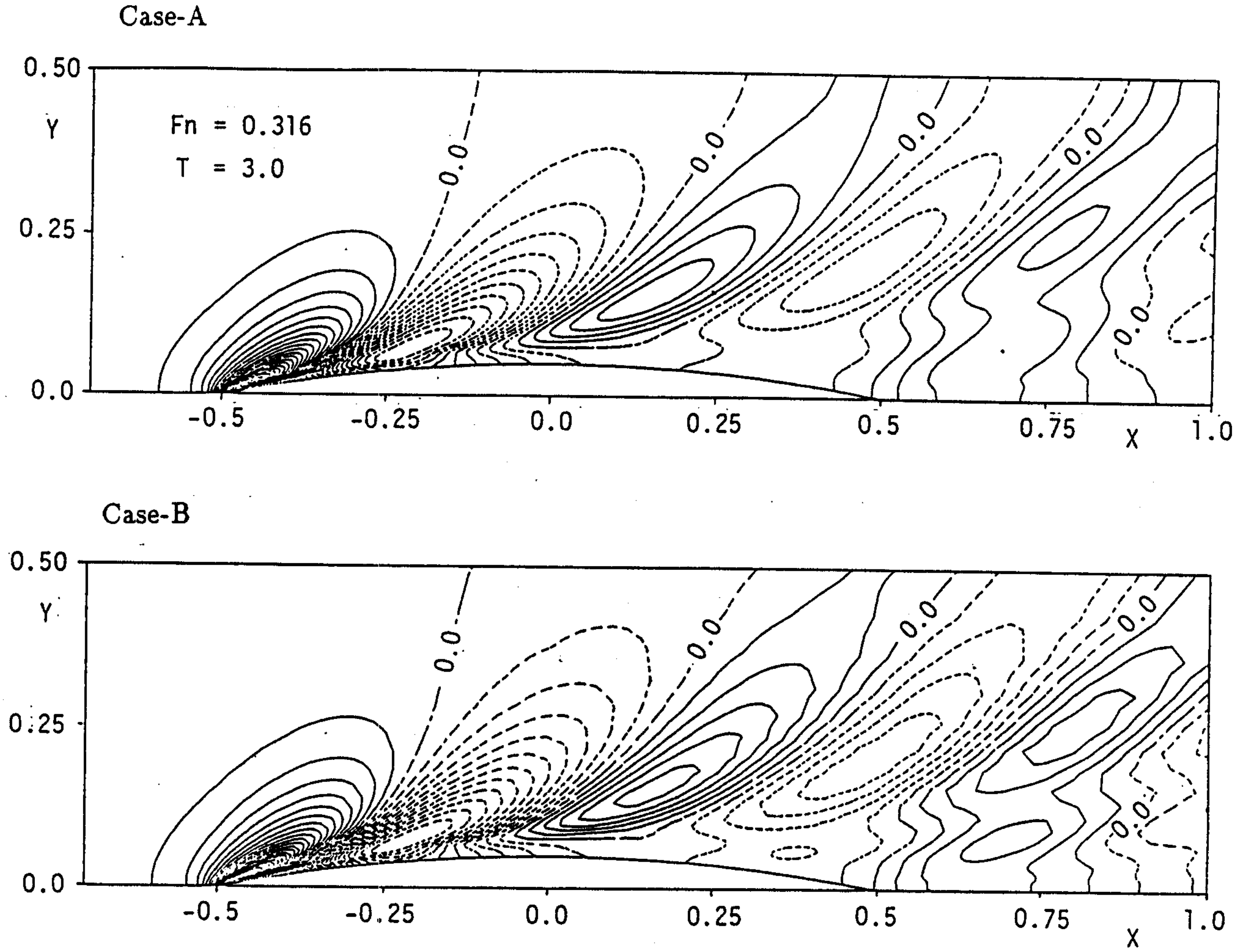

Fig.1 Comparison of wave patterns of Wigley Model

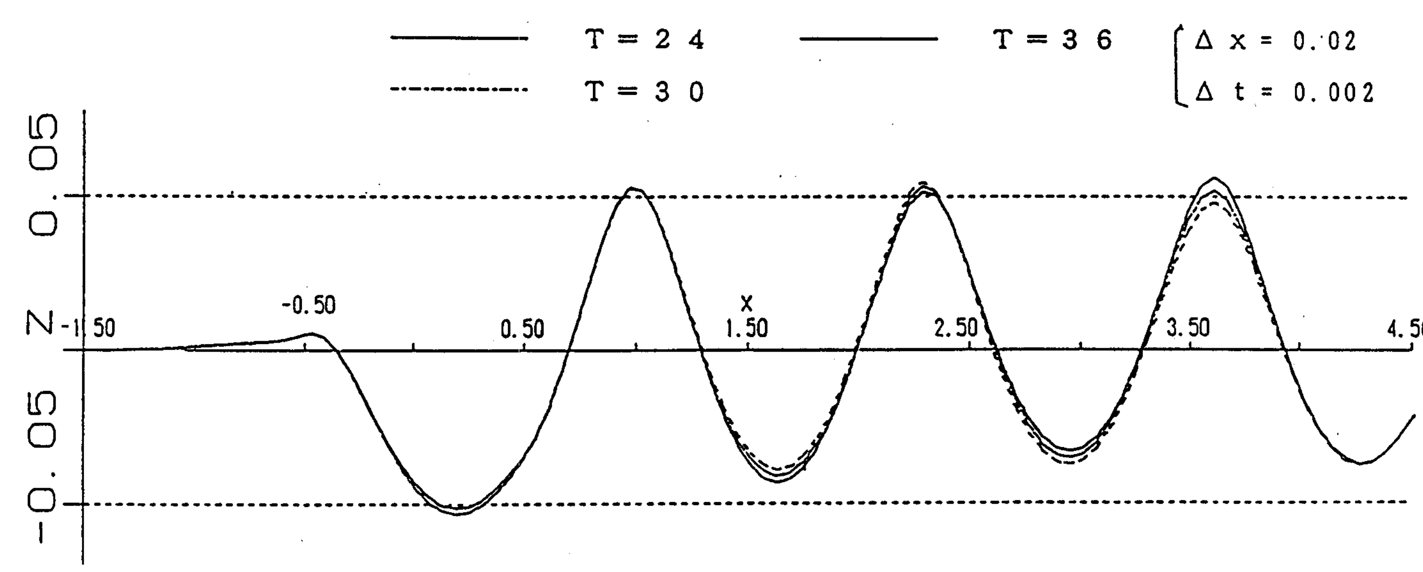

Fig.3 Time history of wave development

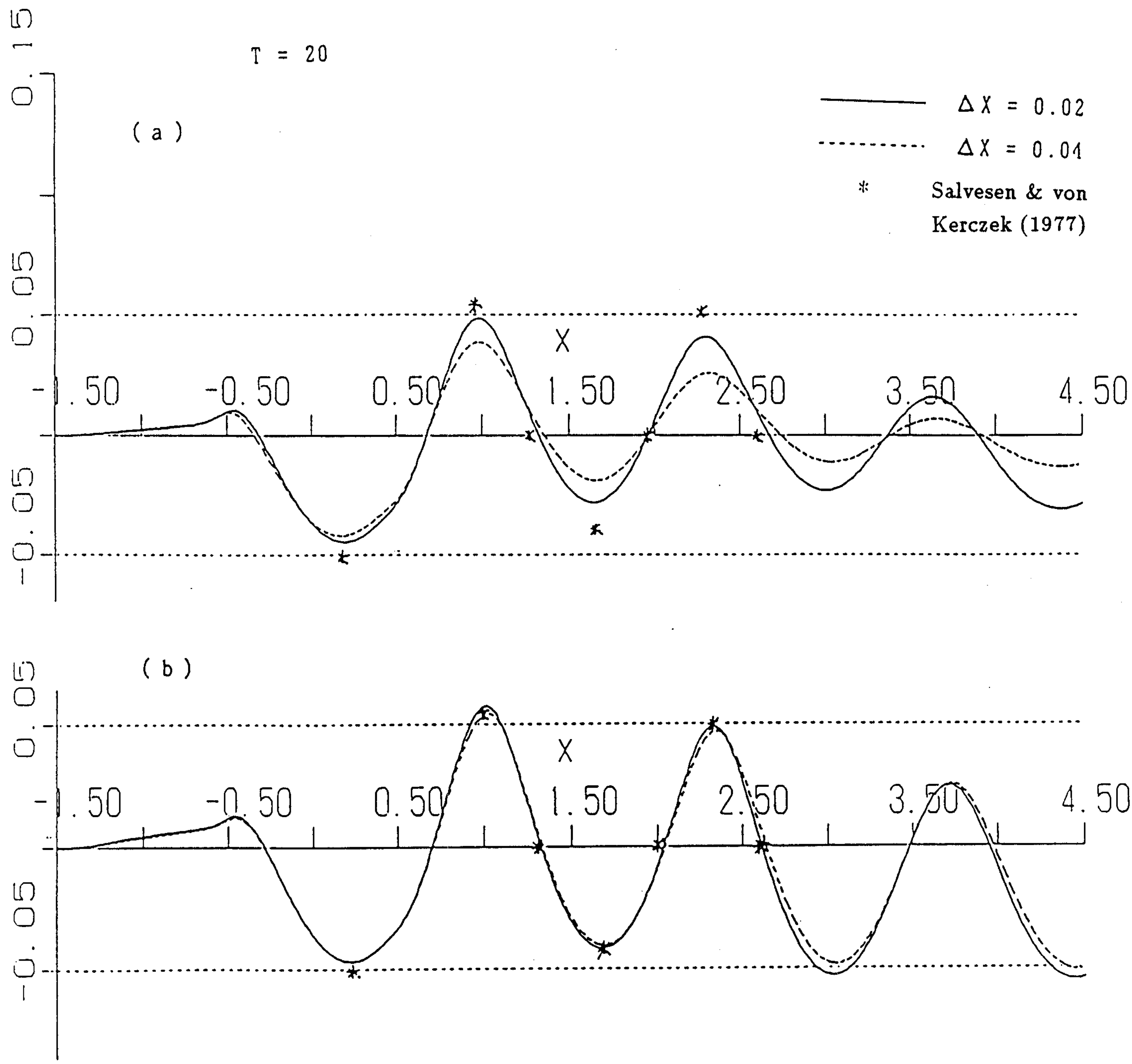

Fig.2 Comparison of wave development by the pressure

DISCUSSION

TUCK: When you compute solutions of the Navier-Stokes equation for the Wigley hull at Re=10^4, how well does your grid capture the boundary layer?
MORI: Usually ten grids are required for the resolution of the boundary layer flow, at least. In our computation the minimum space is 0.002L for the Wigley hull at Re=10^4. For the turbulence case the wall function is used.

SCHULTZ: In your abstract you indicate that the MAC method is first order accurate in space. A more complete implementation scheme can have arbitrarily high-order accuracy. I am also interested in how effective your wave absorber is although $\frac{\partial W}{\partial x}$ is discontinuous there. Can you comment?
MORI: Our experience is limited to the first order accurate. It may be true that the accuracy may be improved by employing a higher order scheme, but the development of the wavy surface may not change so much. The third derivative accelerates the propagation.
Our numerical experiments tells that smooth change of W in x-direction works better, although discontinuous change of $\frac{\partial W}{\partial x}$ is unavoidable.

YEUNG: I think a more thorough test of your absorber condition is to treat problems that are not highly convective. As an example you may like to study the reflection characteristics of your absorber as a function of wave frequency. You mentioned the nonlinear solution of Salvesen and van Kercek (1981). I believe that was 1977 and was a numerical solution not an analytical solution.
MORI: So far we have not studied as a function of wave frequency. But we studied the effects of the length and the distribution of the suppression to find that there is an optimum. This may support your suggestion. I should not forget the year of the 2nd conference when I visited US for the first time and was impressed. I should check the content of their paper more carefully.

GREENHOW: You stated that the translation of a sphere below the free surface had been solved analytically. Please give a reference?
MORI: M. Bessho, "On the Wave Resistance Theory of a Submerged Body," 60th Anniversary Series, Soc. of Naval Arch. of Japan, Vol. 2, pp. 135-172 (1957).