NUMERICAL MODELING OF PROGRESSIVE WAVE ABSORBERS

B. MOLIN
Institut Français du Pétrole

J-M. FOUREST
Ecole Centrale de Nantes

So-called progressive wave absorbers, consisting in a series of perforated vertical screens, have become a substitute to the usual damping beaches in wave tanks. Optimization of the number of screens, spacings, and porosity ratios, is no easy matter and has been apparently based, so far, on extensive model testing [1].

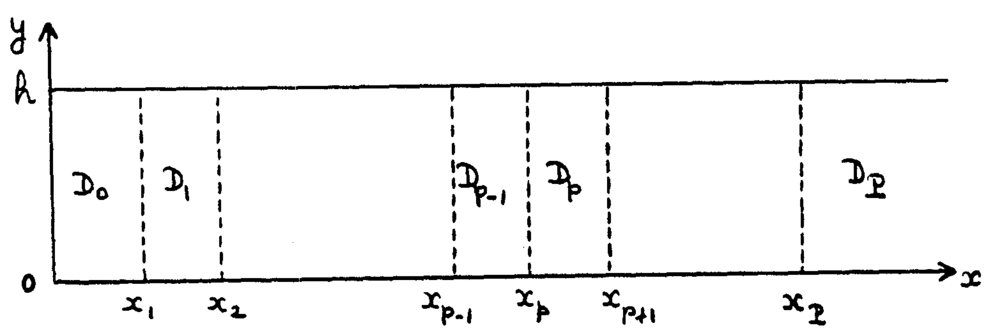
A theoretical modeling of such absorbers was presented by Evans [2], who assumed linear potential theory to hold in the fluid sub-domains inbetween the screens, and the screens to create **linear** losses of head (pressure differential proportional to the traversing velocity). From our experience with slotted and perforated cylinders [3] [4], it seems to us that a **quadratic** relationship is more appropriate. More precisely, we assume here the following law:

$$\Delta p = \rho \, \frac{1-\tau}{2\mu\tau^2} \, U \, |U| \tag{1}$$

where Δp is the (linearized) pressure differential, τ the porosity ratio (defined as the open area divided by the total area), μ a discharge coefficient (depending on the openings geometry and Reynolds number, but close to 1), and U the traversing velocity (assumed to be horizontal).

We present here some preliminary results obtained with that idealization.

The geometry of the problem is sketched below:



P screens are located at abscissas x_p, $(p = 1, \ldots, P)$, from a vertical wall at $x = 0$. The incident regular wave potential is coming from the right with amplitude a and angular frequency ω. We assume a steady state to have been reached, so that the total velocity potential may be written:

$$\Phi(x,y,t) = \Re\left\{\varphi(x,y)\, e^{-i\omega t}\right\}$$

In each fluid sub-domain D_p we write the velocity potential as:

$$\varphi_p(x,y) = \frac{ag}{\omega}\,\frac{\cosh k_0 y}{\cosh k_0 h}\,\left(a_p\, e^{ik_0 x} + b_p\, e^{-ik_0 x}\right) + \frac{ag}{\omega}\sum_{n=1}^{N} \cos k_n y \,\left(c_{pn}\, e^{k_n(x-x_{p+1})} + d_{pn}\, e^{-k_n(x-x_p)}\right)$$

where h is the waterdepth and k_0, k_n are the wave numbers given by:

$$\omega^2 = g\,k_0\,\tanh k_0 h = -g\,k_n\,\tan k_n h$$

In the far right semi-infinite sub-domain D_P we have:

$$b_P = 1 \qquad\qquad c_{Pn} = 0 \quad (n = 1, \ldots, N) \tag{2}$$

In the far left one (D_0):

$$a_0 = b_0 \qquad\qquad c_{0n}\, e^{-k_n x_1} = d_{0n} \quad (n = 1, \ldots, N) \tag{3}$$

The conditions that remain to be satisfied are the conditions at the screens:

• equality of the horizontal velocities, which yields:

$$a_{p-1}\, e^{ik_0 x_p} - b_{p-1}\, e^{-ik_0 x_p} = a_p\, e^{ik_0 x_p} - b_p\, e^{-ik_0 x_p} \tag{4a}$$

$$c_{p-1n} - d_{p-1n}\, e^{-k_n (x_p - x_{p-1})} = c_{pn}\, e^{k_n (x_p - x_{p+1})} - d_{pn} \tag{4b}$$

• the quadratic discharge equation, which we linearize in time:

$$\varphi_{p-1} - \varphi_p = -\frac{4i}{3\pi\omega}\, \frac{1-\tau_p}{\mu_p\, \tau_p^2}\, \varphi_{px}\, \|\varphi_{px}\| \tag{5}$$

Because of nonlinearity, we shall be looking for an iterative resolution method. Assuming the right hand side of this equation to be known, be $f_p(y)$, gives:

$$(a_{p-1} - a_p)\, e^{ik_0 x_p} + (b_{p-1} - b_p)\, e^{-ik_0 x_p} = I_{p0} \tag{6a}$$

$$c_{p-1n} - c_{pn}\, e^{k_n (x_p - x_{p+1})} + d_{p-1n}\, e^{-k_n (x_p - x_{p-1})} - d_{pn} = I_{pn} \tag{6b}$$

with:

$$I_{p0} = \frac{\omega \cosh k_0 h}{a g}\, \frac{\int_0^h f_p(y) \cosh k_0 y\, dy}{\int_0^h \cosh^2 k_0 y\, dy} \qquad\qquad I_{pn} = \frac{\omega}{a g}\, \frac{\int_0^h f_p(y) \cos k_n y\, dy}{\int_0^h \cos^2 k_n y\, dy}$$

Combining equations (4) and (6) gives:

$$\begin{aligned} a_p &= a_{p-1} - \frac{1}{2} e^{-ik_0 x_p}\, I_{p0} \\ b_{p-1} &= b_p + \frac{1}{2} e^{ik_0 x_p}\, I_{p0} \\ c_{p-1n} &= c_{pn}\, e^{-k_n (x_{p+1} - x_p)} + \frac{1}{2} I_{pn} \\ d_{pn} &= d_{p-1n}\, e^{-k_n (x_p - x_{p-1})} - \frac{1}{2} I_{pn} \end{aligned}$$

Since $b_P = 1$ all b_p may be obtained and then all a_p since $a_0 = b_0$. Similarly all c_{pn} and d_{pn} are obtained from the end conditions. This provides new values, be $\tilde{a}_p, \tilde{b}_p, \tilde{c}_{pn}, \tilde{d}_{pn}$, as functions of the $a_p^{(j)}, b_p^{(j)}, c_{pn}^{(j)}, d_{pn}^{(j)}$ at the previous iteration. We then take as old values at the following iteration:

$$a_p^{(j+1)} = a_p^{(j)} + \zeta\, (\tilde{a}_p - a_p^{(j)})$$

$$b_p^{(j+1)} = b_p^{(j)} + \zeta\, (\tilde{b}_p - b_p^{(j)})$$

etc.

where ζ is a given coefficient, between 0 and 1, small enough that the iteration scheme eventually converge.

In the case of small porosities, it has been found more efficient to select another scheme, by reversing eq. (5):

$$\varphi_{px} = i\sqrt{\frac{3\pi\,\omega\,\mu_p\,\tau_p^2}{4(1-\tau_p)}}\;\frac{\varphi_{p-1}-\varphi_p}{\|\varphi_{p-1}-\varphi_p\|^{\frac{1}{2}}} = g_p(y)$$

In the final model, both schemes have been implemented, some screens being considered as lightly porous (and solid at iteration nb. 0) and following the second scheme, and the other ones as quite open (and non-existent at iteration nb. 0), and following the first scheme. The choice between either category is decided upon the value of the parameter:

$$\frac{4\,k_0\,a}{3\pi\,\tanh k_0 h}\;\frac{1-\tau_p}{\mu_p\,\tau_p^2}$$

Numerical results given by the model were first compared with Jamieson and Mansard's experimental results [1] (Figure 1). All μ_p were taken equal to one. The agreement appears to be reasonably good except at low porosities where substantial discrepancies occur. These probably are due to the fact that the screen openings are not horizontal, but slanted, so that the "effective" porosity is somewhat larger than used in the calculations (Jamieson and Mansard only give the "horizontal" porosity). Note that the performance of the absorber notably depends on the wave steepness. This is a convincing proof that a linear discharge law does not apply.

Further insight in the physical phenomena was gained through comparisons with T. Faure's experimental results [5] for one screen only with no reflective wall. It is straightforward to show, from the equality of the horizontal velocities at the screen, that in such case the (complex) reflexion and transmission coefficients verify:

$$R + T = 1$$

Hence one should have:

$$\|R\| + \|T\| \geq 1 \tag{7}$$

Figure 2 shows typical results, where it can be seen that the agreement between theory and measurement is reasonably good at small wave steepnesses, but that it quickly degrades as the steepness increases. Further the experimental results do not verify eq. (7). This suggests that other energy dissipation processes are at hand, most probably wave breaking at the free surface. That such breaking should occur (and can be seen on photographs) is no surprise since the theory predicts the water levels to be different on either side of the screen.

Last we show on Figure 3 some free surface plots for a 15 screen absorber. Increasing the number of screens has the result of diminishing the heights of the "steps" at the screens so that the relative importance of breaking in the dissipation process should decrease correlatively.

REFERENCES

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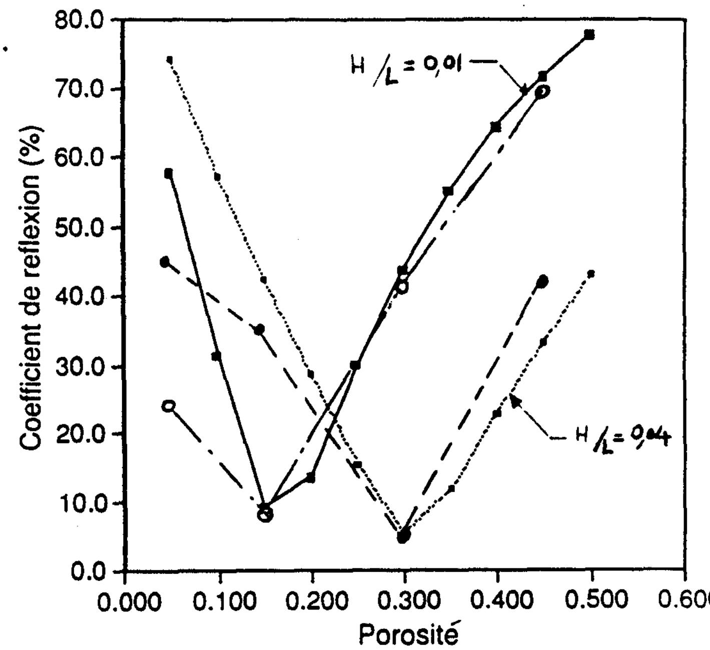

Fig. 1: 5 plates of equal porosities and spacings (30cm). Reflexion coefficient versus porosity.
$h = 1$ m $\quad T = 2.16$ sec.
$H/L = 0.01$: comp. val. meas. val.
$H/L = 0.04$: comp. val. meas. val.

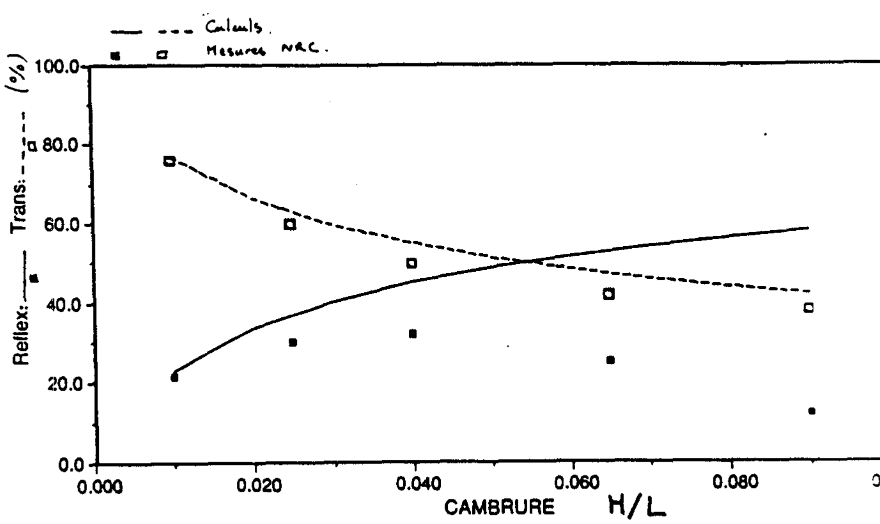

Fig. 2: one plate in an infinite channel
$h = 1$ m; $T = 1.54$ s; $\tau = 10$ %
Reflexion and transmission coefficients vs. wave steepness.

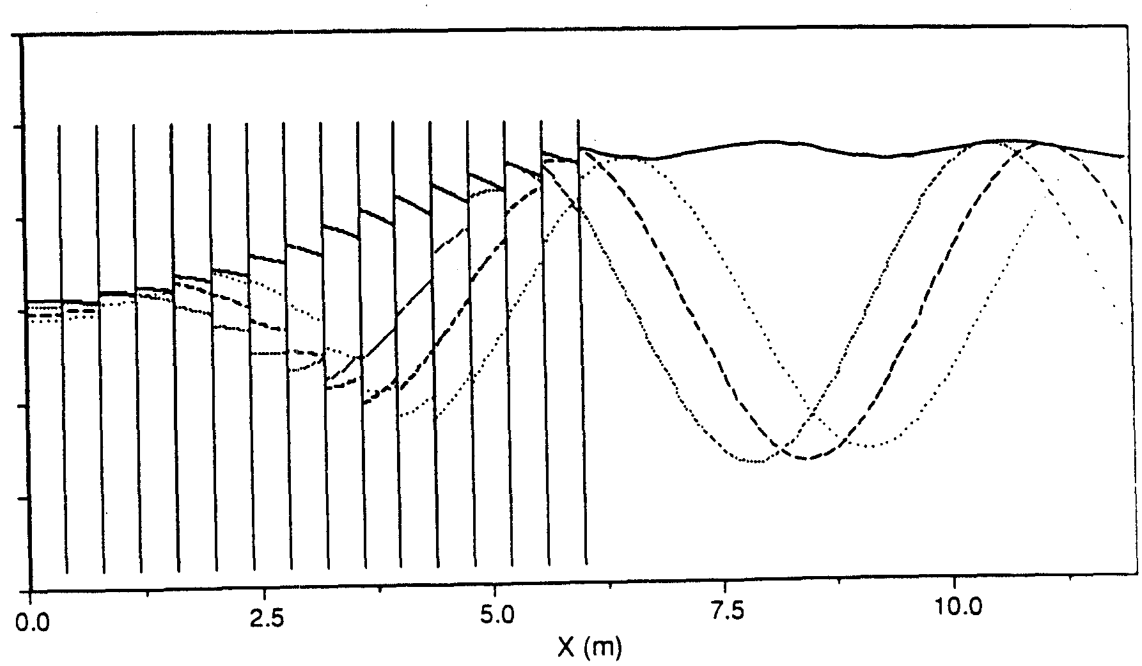

Fig. 3: 15 plates absorber
$h = 1$ m; $T = 2$ s; $k_0a = 0.2$
—— free-surface elevation envelope
------ free-surface profiles at different instants.

DISCUSSION

YUE: Your figure 1 is only at 1 frequency and 2 porosities and 2 wave steepnesses (for which there seems to be great sensitivity). Can you please comment on (or give more results) for the frequency bandwidth and wave steepness/porosity dependences for the absorption of the progressive wave absorbers?
MOLIN & FOUREST: More results can be found in Jamieson and Mansard's paper.

SCHULTZ: For the expanded metal absorbers, it would seem to be a simple extension to your model to have the effective porosity of the plate to be a function of the local velocity orientation. Perhaps this could model some of the preferred orientation/anisotropy questions.
MOLIN & FOUREST: I assume that you refer to the fact that, with expanded metal sheet, the openings are not horizontal but slanted. It is no problem to introduce their inclinations in the numerical model. We have not attempted to do it.

GRUE: Another kind of wave absorbers is an arrangement of wedge forms with the length direction along the propagating wave direction. This is a kind of vertical beaches, instead of horizontal, and is used by Steven Salter at Edinburgh University. What is your comment about such an arrangement. Can you perform computations for this geometry?
MOLIN & FOUREST: Thank you for mentioning this other system I did no know about. It would require quite a different kind of modeling.

TULIN: A am not sure what's wrong with a beach! We have found a porous beach (honeycomb) inclined at a small angle to the horizontal to be very effective.
MOLIN & FOUREST: I am not advocating for progressive absorbers against beaches. Each system probably has its own advantages. Progressive absorbers advantages are that they are quite insensitive to the water level and that their overall lengths are shorter than beaches.