NONLINEAR BOW FLOWS WITH SPLASHES

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1 Introduction

The inviscid, irrotational flow past the bow of a two-dimensional ship moving at a constant velocity is considered. Numerical computations taking into account the full nonlinearities of the boundary conditions on the free surface are still almost non-existent. It is well known that most of the wave resistance is caused by the waves breaking at the bow.

Several configurations have been proposed for bow flows (see for example figure 1 in Dagan and Tulin (1972)):
(1) The free surface approaches smoothly the bow,
(2) A splash appears near the bow,
(3) A jet rises along the bow and the returning jet is neglected.

Let the Froude number be defined as

$$F_d = \frac{U}{\sqrt{gd}},$$

where U is the velocity of the ship, g the acceleration due to gravity, d the draft. The advantage of this particular Froude number is that it can be used in infinite depth. For type (2) and type (3) flows, another Froude number, F, which will be used below, can be defined by taking the thickness δ at infinity of the jet rising along the bow rather than the draft as the typical length.

Dagan and Tulin (1972) used asymptotics to consider type (1) solutions for small Froude numbers and type (3) solutions for large Froude numbers. They suggested the splash model for intermediate Froude numbers. Fernandez (1981) studied three-dimensional flows past ships of finite length by using matched asymptotic expansions. He found that the bow flow can be described by a sheet of water rising along the bow and falling back far enough to neglect the effect of the re-entry jet. Vanden-Broeck, Schwartz and Tuck (1978) and Vanden-Broeck and Tuck (1977) showed that type (1) solutions are not possible for flat bows in water of infinite depth. Vanden-Broeck (1989) computed bow flows in water of finite depth and showed that there is a range of supercritical Froude numbers for which type (1) solutions are possible. Dias and Christodoulides (1991) computed splash-like flows in a corner, where the whole oncoming flow rises along the wall.

They were able to model the loop that the sheet of water makes as it falls back onto the oncoming flow. They found that such solutions exist only above a critical Froude number. Vanden-Broeck (1992) computed similar flows using finite differences. Dias and Elcrat (1992) considered the infinite Froude number limit, in which the jet keeps rising and does not fall back onto the oncoming flow (type (3) flows). They found a unique solution, which is characterized by $d = \delta$. Grosenbaugh and Yeung (1989) and Yeung (1991) computed fully nonlinear solutions of the unsteady two-dimensional bow-flow problem where there is no splashing .

Here, we study type (2) solutions by solving the complete nonlinear problem in the whole flow domain. Part of the oncoming flow is allowed to rise along the bow. This rising flow must come to rest because of the effect of gravity. Then it forms a jet which falls down to infinity. For simplicity, the jet is assumed not to intersect the oncoming flow.

2 Formulation of the problem

The steady irrotational flow of an incompressible inviscid fluid past the bow of a ship is considered (see figure 1). The ship is assumed to be semi-infinite. For simplicity, the bow is assumed to be vertical for the formulation. In a frame of reference moving with the ship, we study the problem of a uniform flow approaching the ship from the right with velocity $-U$. The water is supposed to be of infinite depth. We denote infinity by I, the stagnation point at the intersection of the bow and of the free surface by S, the point where the dividing streamline ends by D and the corner of the ship by C. As the flow approaches the bow, part of it rises along the bow and eventually comes to rest. It then becomes a jet which falls down to infinity (point J). It is assumed here that the jet does not cross the oncoming flow. The problem is nondimensionalized by taking U as the unit velocity and the upstream thickness δ of the jet as the unit length. In dimensionless coordinates, Bernoulli's equation on the free surfaces takes the form

$$u^2 + v^2 + \frac{2y}{F^2} = 1$$

if we choose the x-axis to be along the free surface at infinity. As the bow is assumed to be vertical, the y-axis is chosen along the bow.

We denote the velocity potential by $\phi(x,y)$ and the stream function by $\psi(x,y)$. In addition we introduce the complex variables $z = x + iy$ and $f = \phi + i\psi$. The flow domain in the f-plane is shown in figure 2. There is a slit starting at D. The distance between the streamline IJ and the slit (i.e. the flux going into the jet) is 1.

We transform the domain occupied by the fluid in the f-plane into the upper half of the unit disk in the t-plane so that the points S, I and J are mapped into the points -1, 1 and i. The ship goes onto $[-1,1]$. We denote the images of the points C and D by t_c and t_d. The transformation from the f-plane to the t-plane can be written in differential form as

$$\frac{df}{dt} = \frac{4}{\pi(1+t_d^2)}\left[\frac{(t-t_d)(1-tt_d)(1+t)}{(1-t)^3(1+t^2)}\right].$$

We introduce the hodograph variable

$$\zeta(z) = \frac{\mathrm{d}f}{\mathrm{d}z}(z) = u - iv.$$

The problem to be solved is to find ζ as an analytic function of t satisfying Bernoulli's equation on the free surfaces and the kinematic boundary conditions on the real diameter $t \in [-1, 1]$. Points on the free surfaces are represented by $t = e^{i\sigma}$.

We write the hodograph variable ζ as

$$\zeta = -\frac{(t+1)}{2} \frac{(t-t_d)}{(1-t_d)} \frac{(t-t_c)^{-\frac{1}{2}}}{(1-t_c)^{-\frac{1}{2}}} \frac{[-\ln c(1+t^2)]^{\frac{1}{3}}}{[-\ln 2c]^{\frac{1}{3}}} \left[1 + \sum_{n=1}^{\infty} a_n (t^n - 1)\right].$$

With such an expansion, the velocity is automatically -1 at infinity upstream of the ship and has the correct behaviour at the points D, C, S and J. The only condition left to satisfy is Bernoulli's equation on the free surfaces. The constant c is a real number between 0 and 0.5. We truncate the infinite series after $N-2$ terms. For a given t_d, there are N unknowns: $N-2$ coefficients a_n, F and t_c. We choose N collocation points along the free surfaces, satisfy Bernoulli's equation at them and obtain a system of N nonlinear equations, which we solve by Newton's method.

Once the coefficients in the series have been obtained, one can compute the height CS and the elevation of the stagnation point D, and plot the free surfaces.

Numerical results will be presented for various values of the Froude number in both finite and infinite depth. As an example we include a computed bow flow in water of infinite depth for $F = 3.74$ (see figure 1).

The range of Froude numbers for which such solutions exist is being explored presently.

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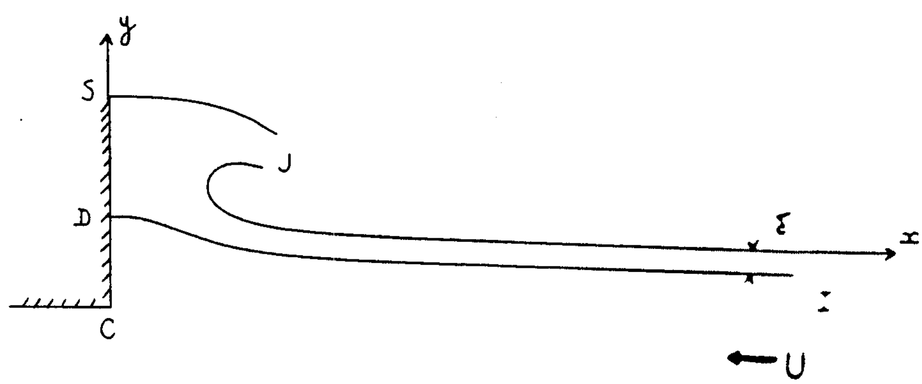

FIGURE 1

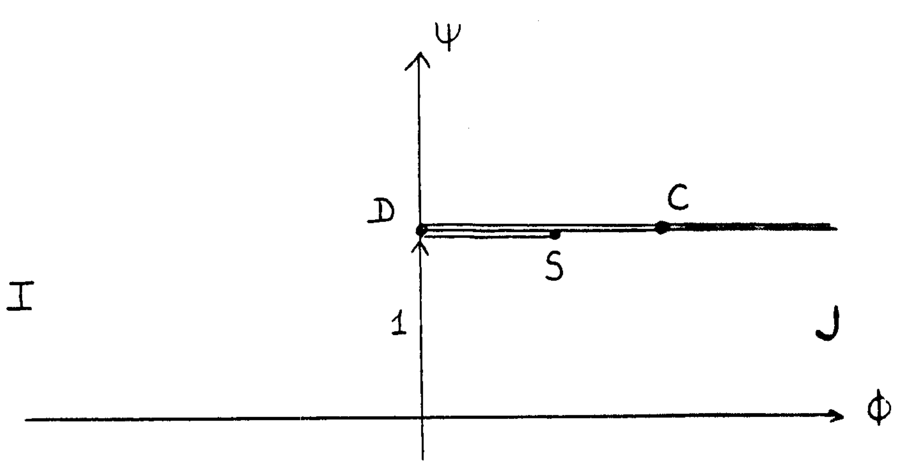

FIGURE 2

DISCUSSION

SCHULTZ: Your figures show the geometry & scale of the breaker to be nearly identical. What could cause this to change — e.g limiting Froude number, surface tension, etc.

DIAS & VANDEN-BROECK: The limit as the Froude number decreases is still under investigation. It is not clear why surface tension might be included in our model.

BECK: How can you have a stagnation point on the free surface?

DIAS & VANDEN-BROECK: The problem we are solving is steady. An Eulerian description is used. Having a stagnation point on the free surface occurs in many problems.

RAVEN: All your calculations concern steady flows. Both from experiments and from certain numerical solutions (e.f. Grosenbaugh et al.) it seems that the 2D flow is unsteady and periodic at least in a certain range of Froude numbers.

Do your results give any indication of non existence or instability of a steady solution? Or would the interference of the returning jet with the incoming flow be an important factor here?

DIAS & VANDEN-BROECK: Of course, the impact of the returning jet has not been investigated and might prevent to reach a steady flow. But we believe our model and solution is a first step towards the description of the splash in front of the bow.

TUCK:

(a) (In partial answer to question by W. Schultz). The results presented are for fixed jet thicknesses, and the draft increases as Froude decreases. Thence they give the (probably correct, but still to be checked) impression that the jet region is becoming of a fixed character.

(b) The results are very recent. I look forward to seeing results as Froude becomes even smaller. I had expected that the jet region would contract toward the stagnation point, but it seems that instead it may be moving upstream away from the body. Meanwhile, it's overall size is decreasing relative to the draft.

TULIN: The problem is very interesting and instructive. With regard to other work:

(1) Fernandez used inner-outer expansions to correct the 2D solution locally, with the 3D ship (flat hull) solution far away;

(2) Grosenbaugh and Yeung, and Susan Cole observed the intermediate Froude flow to involve an eddy sitting on the dividing streamline. The solution with the stagnation point or the free surface might eventually teach us why the search for a smooth bow flow at low Froude failed (Tuck, Vanden-Broeck, Shwartz). As I recall, they could find a solution if they allowed a jump in the free surface, upward as the bow is approached.