Computation of Nonlinear Waves Generated By Floating Bodies

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1 Introduction

In this study, the computation of the nonlinear waves generated by a floating body is attempted using a desingularized method. The method has been successfully applied to several problems involving submerged bodies or singularities (Cao *et al* 1990 and Cao 1991). For floating bodies, there are several additional difficulties associated with the method. The first is the ability of the desingularized method to handle a discontinuity of the unit normal to the boundary. The second is the difficulty associated with the conflict of boundary conditions along the intersection of the free surface and the body surface. The third is the localized wave breaking in the fully nonlinear calculations. As discussed in the following sections, we are presently examining these problem areas.

2 Formulation and Solution Procedure

The fluid is assumed inviscid and the flow irrotational so that the velocity potential exists. The kinematic condition on the wetted body surface and the kinematic and dynamic boundary conditions on the free surface need to be satisfied. An initial value problem starting from rest is assumed so that a radiation condition is unnecessary. The problem is solved in the time domain by a time stepping procedure. At each time step, the free surface and body surface positions are known. The potential on the free surface and the normal velocity on the body surface are also known. Thus, a boundary value problem (BVP) can be solved to determine the value of the velocity on the free surface and the potential on the body. Then, the free surface potential and position are updated by time integration of the free surface kinematic and dynamic boundary conditions. This procedure repeats itself at the next time step.

The BVP is solved using the desingularized method in which the velocity potential is constructed by a distribution of isolated sources inside the body and above the free surface. The source strengths are determined so that the Dirichlet condition on the free surface and the Neumman condition on the body surface are satisfied at chosen collocation points. To update the free surface position, several numerical techniques are possible. The first is a Lagrangian approach where the collocation points follow the same material points whose positions are updated during the whole period of the computation. For problems with forward speed or large amplitude motions, Lagrangian collocation points can pile up or penetrate into the hull near the bow region. To avoid this, the collocation points need to be regrided when necessary and a very small time step may be required. Another approach is to use Eulerian collocation points which are constrained to move only in the vertical direction. However, the Eulerian approach requires the numerical spatial derivatives of the wave elevation, which usually results in large non-physical reflection from the open boundaries. A generalized collocation point approach has been proposed. In this approach, the horizontal motion of the collocation points are prescribed while the vertical motion is updated by time stepping. The horizontal motion of the collocation points is chosen close to the incoming uniform flow so that the errors induced by the numerical evaluation of the free surface spatial derivatives are greatly reduced. A combined use of Eulerian, Lagrangian and the generalized collocation points is used in the free surface updating. More details are given in Beck & Cao (1993).

3 Bodies With Sharp Edges

In the desingularized method, numerical difficulties may occur in the vicinity of the a sharp edge. One of the difficulties is due to the discontinuity in the unit normal of the surface. The other is that the singularity distribution may cross over the centerline or even the body surface on the other side since the desingularization distance is proportional to the local surface grid size (Cao *et al*, 1991). Usually, these types of difficulties can be avoided by careful discretization and desingularization, at times, it may be necessary to reduce the grid spacing and the desingularization distance (Han *et al*, 1987 and Johnston *et al*, 1984). To examine this problem, we first investigated the flow past a 2-D Karman-Trefftz airfoil (Milne-Thomsom, 1973) for which the exact solution is known. The Karman-Trefftz airfoil is a concave section with sharp leading and trailing edges.

Several choices of the discretization (therefore desingularization) were tested. The isolated singularities were placed inside the body along the normal direction from the surface grid points. The desingularization distance was proportional to the local grid size. Near the leading and trailing edges, the desingularization distance was reduced so that the singularities were located on the centerline to avoid the cross over of the singularities beyond the centerline.

The desingularized method was tested using both uniform and cosine spacing in the longitudinal direction for the collocation grid on the body. The calculations showed that the cosine spacing was preferable to the uniform spacing. The results in figures 1 and 2 were computed using cosine spacing. These figures show the comparisons of the numerical results to the exact solution for the tangential velocity on body surface. For the results presented in Fig.1, the total velocity at the sharp leading and trailing edges(stagnation points) was constrained to be zero. As can be seen, the comparison is very good for most of the body length except near the edges where a spike develops. Fig.2 shows the results with the collocation points at the stagnation points removed. This eliminated the spikes. Apparently, the spikes were due to the use of the sharp edges as collocation points where the numerical technique could not resolve the extreme (jump) change in the unit normal direction.

The method has also been tested with a 3-D double-body Wigley hull and the same conclusion was drawn: collocation points can not be located at the sharp bow and stern without causing spikes in the solution.

4 Oscillating Cylinder

Computations have been carried out for a right-circular cylinder of diameter B, bottom depth T, and with its centerline oriented in the vertical direction. Starting from an initial state of rest at time t=0, the body is forced to periodically move vertically or horizontally with a velocity

$$u(t) = \begin{cases} 0 & t \leq 0 \\ -\omega \; a \; sin(\omega t) & t \geq 0 \end{cases}$$

For these calculations, the position of the free surface was determined using the Lagrangian approach with regriding as necessary.

Wave computations for the heave and sway motion of a 2-D box have been done. In the computations, the intersection points were constrained to move along the body surface. For heave, there were no apparent difficulties. For large sway motion, the collocation points tended to pile up near the intersection points and regriding was necessary. Fig. 3 shows the time histories of the free surface profiles due to the forced heave motion of the 2-D box. The time is increasing from the bottom to the top of the figure. The comparisons of the added mass and damping coefficients to the linear calculation and experiments by Vugts (1968) were good. The results for 3-D computations need to be confirmed.

5 Modified Wigley Hull With Forward Speed

The problem of a modified Wigley hull (*cf.* Journée, 1992, Modal I) starting from rest and smoothly accelerating to a constant forward speed is presently being investigated. A coordinate system moving with the ship is used. The velocity potential is first decomposed into,

$$\Phi(\vec{x},t) = U_o(t)x + \phi_w(\vec{x},t)$$

where $U_o(t)$ is the ship speed and $\phi_w(\vec{x},t)$ is the disturbed potential field due to the presence of the ship and the free surface. In our computation, $\phi_w(\vec{x},t)$ is constructed as

$$\phi_w = \sum_{j=1}^{N_f} \sigma_j^f \; G_j^f + \sum_{k=1}^{N_b} \sigma_k^b \; G_k^b$$

where G_j^f and G_k^b are the potentials of the isolated sources above the free surface and inside the body respectively. σ_j^f and σ_k^b are the source strengths to be determined by the boundary conditions. The free surface is updated by a combined use of Eulerian, Lagrangian and generalized collocation points. Both the body and free surface boundary conditions are forced to be satisfied at the collocation points along the intersection line. At the upstream edge of the computational domain, the wave elevation and $\nabla\phi_w$ are set to zero. Fig. 4 shows the wave generated by the modified Wigley hull for Froude number $F_r = 0.214$ where the ship has traveled about a distance of 4.5 times its length.

Presently, the computation must be stopped after the hull has traveled approximately half to one of its length for higher Froude numbers. The problem is at the bow where the water surface has a large amplitude and very steep slope as a thin sheet rides up the side of the hull. This may be the initial formation of a spray sheet and breaking bow wave. Physically, this phenomenon can be observed in model tests and at full scale. The problem is to find a local solution that properly accounts for this behavior while at the same time not disrupting the computations for the entire domain. This difficulty only occurs in the fully nonlinear calculations where the free surface position is determined as part of the solution procedure.

6 Reference

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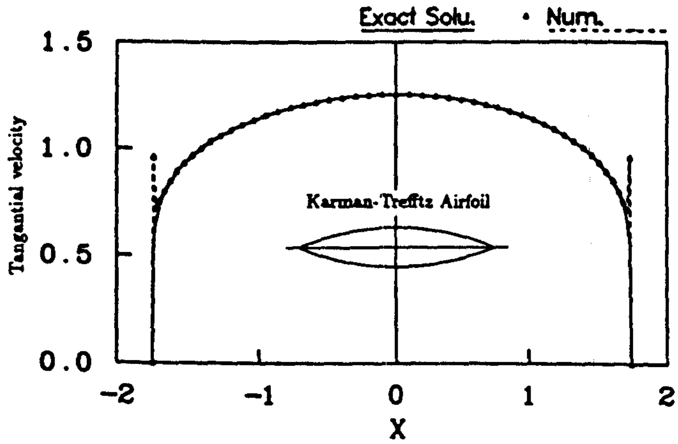

Figure 1 Tangential velocity on the body surface. Collocation points at stagnation points in the numerical result.

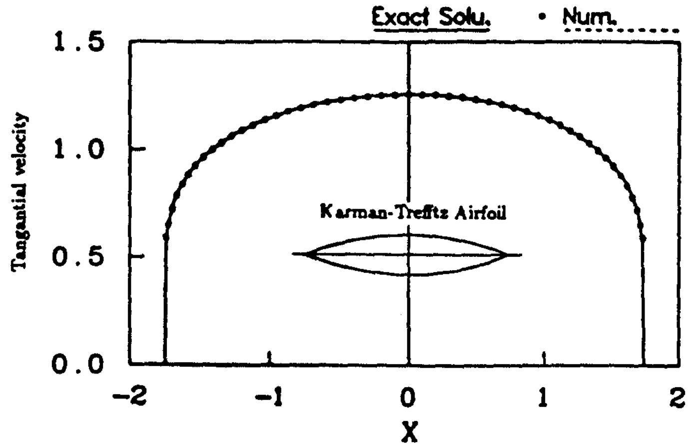

Figure 2 Tangential velocity on the body surface. Collocation points not at stagnation points in the numerical result.

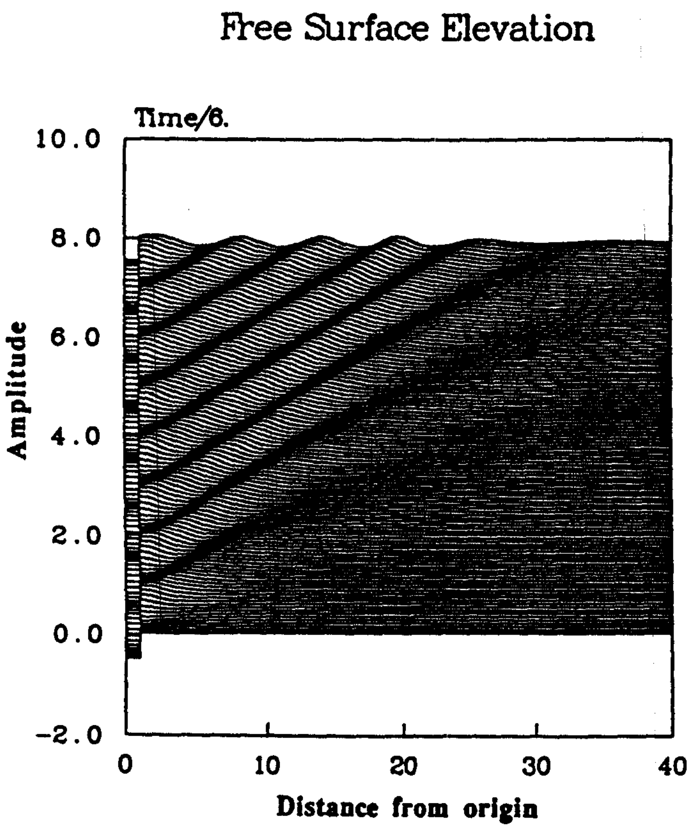

Figure 3 Fully nonlinear waves caused by the vertical oscillation of a rectangle with B/T=1. The amplitude of oscillation is .1B and the frequency is $\frac{\sigma^2 B}{g} = \pi/3$.

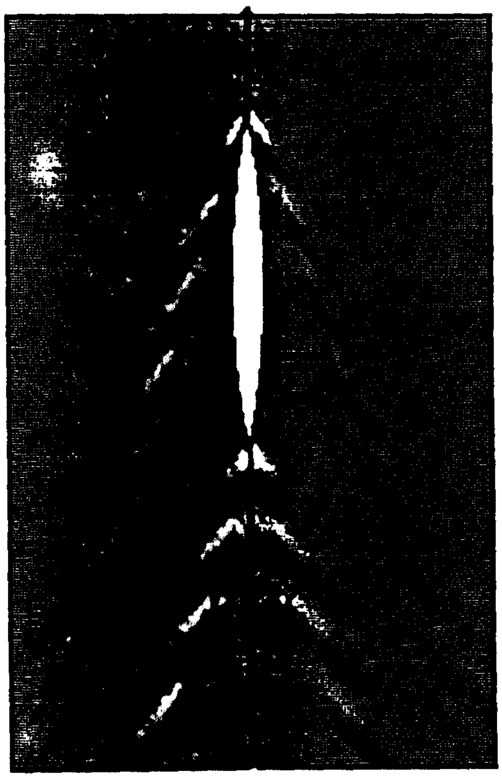

Figure 4 Wave pattern of Wigley hull; ($F_r = U_o/\sqrt{gL} = 0.214$, and The ship has traveled a distance of 4.5 times its length).

DISCUSSION

Bertram: How did C_w compare to experiments for the Wigley-hull?

Cao, Lee and Beck: We haven't yet completed the computation for the forces acting on the body (drag C_w, lift and moment). Since the problem is unsteady, the time derivative of the velocity potential $\frac{\partial\phi}{\partial t}$ and the change of the wetted hull surface with time must be taken into account in the integration of the pressure on the hull. In addition, there is a problem in computing the pressure because $\frac{\partial\phi}{\partial t}$ along the changing intersection line is difficult to compute. An alternative is to exchange the order of the integration over the wetted hull and the differentiation of the velocity potential in the force calculation using Lipniz's rule. The computer code is presently being developed.

Jensen: On the slice where you introduce desingularization it appears that you have a collocation point at the body-water intersection. Is that true? Do you apply two boundary conditions there?

Cao, Lee and Beck: Yes, we have a collocation point at the body-water intersection and apply two boundary conditions, the body boundary condition and the free surface Dirichlet condition.

Martin: In Milne-Thomson's notation, the Karman-Trefftz mapping is

$$\frac{z-kl}{z+kl}=\left(\frac{\eta-l}{\eta+l}\right)^k$$

where k is a parameter, satisfying $1 < k < 2$; one maps the circle $|\eta| = l$ into the shape shown in Fig. 1, with corners at $z = \pm kl$. What value of k did you use?
For uniform flow in the x-direction, the (complex) velocity near $z = \pm kl$, say, is proportional to

$$(z-kl)^\nu$$

where

$$0 < \nu < (2-k)/k < 1.$$

So, unless $k = 4/3$, you do not have a square-root zero at the corner, and so cosine spacing may not be appropriate. Comment?

Miloh: If I remember correctly the solution for the Karman-Trefftz shape near the corner behaves like $x^{\alpha/\pi}$, where $x = 0$ at the apex and $\alpha \neq 0$ is the opening angle. Did you compare your numerical solution agaist the exact formula (higher derivatives as well)?
For 2-D shapes with sharp corner, one usually can not use a brute-force method (arbitrary spacing) near the corner and it is preferable to take advantage of this local solution.

Cao, Lee and Beck: We would like to response to Prof. Miloh and Prof. Martin together since their questions and comments are closely related. We use $k = 2 - 1/4 = 1.75$, which gives an opening angle of $\pi/4$ at the corner. The flow velocity near the corner behaves like $(z-kl)^{\frac{1}{4}}$. We compare the tangential velocity along the body surface to the exact solution (shown in figures 1 and 2). The comparisons for the pressure coefficient are similar to the velocity curves given in Figures 1 and 2. We didn't compare the higher derivatives which we were not interested in. Regarding the grid spacing in the numerical calculation, we agree that the cosine-spacing (or equal-spacing) may not be optimal (for a general solution behavior, it is not easy to find the optimal grid spacing). In this problem, however, since the velocity is never singular in the solution, the difficulty for a numerical method mainly comes from the discontinuity of the surface normal at the corner. The question is whether and how a numerical method can give a good approximation near the corner. Our tests indicate that the desingularized method can give good approximations to the solution. The tests have also shown that whether the stagnation points are used as collocation points or not is more critical than the grid spacing.

We think Prof. Miloh's suggestion of taking advantage of the local solution may improve the performance of a numerical method.