THE DEVELOPMENT OF A THREE-DIMENSIONAL PANEL METHOD FOR NONLINEAR FREE SURFACE WAVES

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Introduction

Boundary integral equation methods are very useful for hydrodynamic (nonlinear) wave problems in domains with arbitrary bottom geometries. Also their application to problems with fixed or freely floating bodies is very promising.

A number of successful two-dimensional methods has been described in literature.

Extending the 2-D methods to three dimensions is possible for most of the methods. However, in practical computations, large problems may occur. The problems are mainly due to the low order of accuracy of the method, the large amount of CPU time needed for the computations, and the complex algorithm for the geometric description and the evolution of the boundary grid. These are the main reasons why to our knowledge only few computations on nonlinear waves with a 3-D boundary integral equation method have been described in literature.

Romate [3] has developed an accurate higher order 3-D panel method for the description of free surface gravity waves. His program is extremely fast due to the high degree of vectorization (for doing a time step in a problem with 650 panels, he needed about 4 CPU seconds on a CRAY-XMP). The method is based on Green's third identity for solving Laplace's equation. For the discretization of the boundary integral equation, the boundary of the domain is divided into quadrilateral panels, with one collocation point per panel. For the influence coefficients computations, a linear source distribution and a quadratic dipole distribution is assumed. Tangential derivatives are determined by finite difference approximations over the collocation points of a number of adjacent panels. For the time integration, he used the classical fourth order Runge-Kutta method with 'frozen coefficients' (i.e. the influence coefficients were not redetermined at the intermediate time levels).

Romate's method provided stable and very accurate results for linear and mildly nonlinear problems. However, in the computation of highly nonlinear waves, instabilities occurred, due to numerically excited cross waves.

In order to improve the results of Romate's method for highly nonlinear wave problems, we have done some investigations, and we have modified the method. In this paper we will discuss some of the modifications. Due to these modifications, stable and very accurate results are obtained for linear and highly nonlinear wave problems. We will present some results of computations on 3-D problems.

A 2-D version of this method has been extended for freely floating bodies. The extension of the 3-D method for freely floating bodies is now in development.

New approaches in the method

From a variational investigation [2] we concluded that using fixed horizontal positions for the lateral boundaries does not provide a well-posed problem, if the fluid particles have a non-zero horizontal velocity at the intersection of the lateral boundary with the free surface. Using a Lagrangian description for the free surface and the inflow and outflow boundaries provides a well-posed problem. For practical reasons we decided to have the vertical inflow and outflow boundaries travelling in horizontal direction along with the horizontal velocity of the fluid particles at the intersections with the free surface. This approach does not violate the well-posedness of the problem. These theoretical conclusions were confirmed by results of numerical tests.

Because grid redistributions reduce the accuracy of the method, we have chosen to replace the algorithms for the geometric modelling and the evolution of the grid. In the present approach, the geometric modelling is based on the positions of the collocation points, and the evolution of the grid in time is determined by the motion of the collocation points. At the free surface, a Lagrangian description for the motion of the collocation points is chosen.
Because the lateral boundaries may travel in a non-Lagrangian way (except for their intersection with the free surface), special algorithms have been developed for the evolution of the grids on the lateral boundaries and the bottom to keep the boundary grids well connected at the intersections. An adaptive grid motion algorithm has been implemented to ensure that the grid has some desirable properties (such as a dense grid on places with large gradients, and only small curvature of the grid lines over the boundaries).

In our method the collocation points are at the centres of the panels. In the original panel method, splines of the third degree were used for the approximation of the surfaces and the determination of the panel variables. However, Arnold and Wendland [1] have shown that this combination may provide singular integral equations, providing numerical instabilities in the computation on problems with highly curved panels. Using a spline of even degree eliminates the singularities. We have experienced these effects in 2-D computations of a breaking wave on a slope, where numerical instabilities spoil the solution before a jet has developed if splines of an uneven degree are used. These instabilities are not observed if an even spline is used.

Because we have chosen not to use grid redistribution techniques, non-equidistant grid-sizes will occur in the computations. That is why we have introduced a new iterative algorithm for determining the intersections of adjacent networks.
The geometric data for the panels of each network are determined from a projection of the network on a rectangle in the computational domain. This implies that multiply connected smooth boundary parts must be split up into a number of networks if constructions are introduced into the fluid domain. For that aim we have extended the algorithms, so that smooth connections can be determined between the networks, and an arbitrary number of adjacent networks can be connected to one network edge.

Because Romate's time integration method may cause a severe loss of accuracy in the computation of highly nonlinear waves, we have implemented another time integration method.
We have considered a number of time integration methods. One step methods are preferable over multi-step methods, because they can be more easily implemented. Implicit methods have not been chosen for the same reason.
A full implementation of a fourth order Runge-Kutta method implies a large increase of CPU time.
Using a third or fourth order Taylor method is less accurate in our case, because higher order

(up to third resp. fourth order) spatial derivatives are needed then. No accurate approximations for these derivatives can be determined with our method.
As a compromise, we have chosen to implement a 2-stage 2-derivative generalized Runge-Kutta method, where the geometric parameters and the influence coefficients are redetermined at the intermediate time level. This method is very accurate, with a minimal increase of CPU time.

Numerical results

Due to the above described improvements, stable and very accurate results are obtained in highly nonlinear wave computations. As a test case, computations are done on a periodic propagating wave, with wave height 5m, wave length 60m and period 6.55s on 10m water (the exact solution is known from Fourier-theory). The height of this wave is over 80% of the theoretical maximum.
The initial solution for this problem is prescribed. On the inflow boundary, the exact normal velocities are prescribed (simulating a wave maker). Figure 1 shows the solution obtained after 3 wave periods. It can be seen in this figure, that the errors in the elevation are below 0.1m (which is 2% of the wave height). No growing errors or instabilities are observed if this computation is continued.

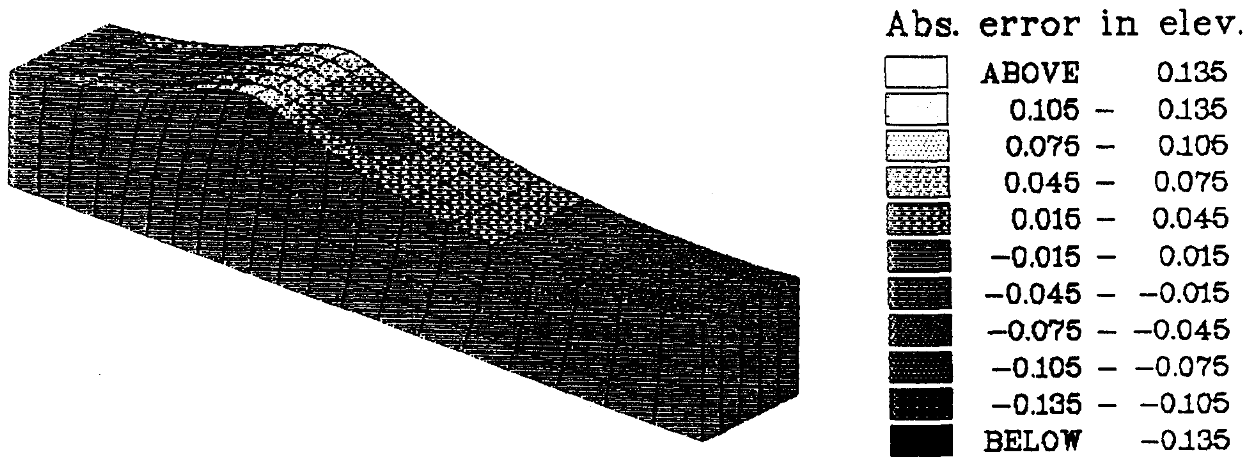

Fig.1. Results of computation on highly nonlinear periodic wave propagating to the right. Shape of the grid and errors in elevation of the solution after 3 wave periods.

Real 3-D effects can be observed from computations on the interaction of a highly nonlinear solitary wave (wave height 3.5m on 5m water) with a construction on the bottom. Fig.2 shows the shape of the free surface and the bottom grid (both including the grid on the lateral boundaries) after the wave has passed over the construction. The results show that the wave becomes higher and steeper above the obstacle, and develops into a breaking wave. Fig.3 gives a better view of the shape of the free surface near the jet (it should be noted that our panel method uses curved panels, but the edges of the panels are represented by straight lines by the graphical software).

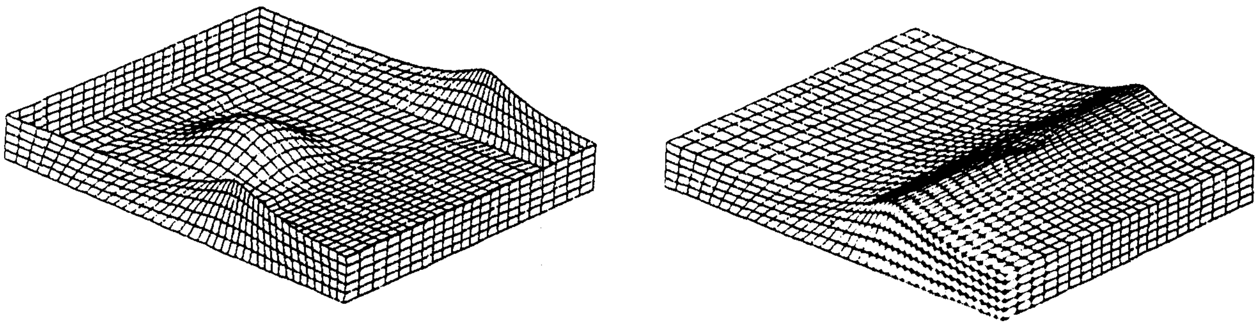

Fig.2. Shape of bottom grid and free surface after interaction of solitary wave with construction.

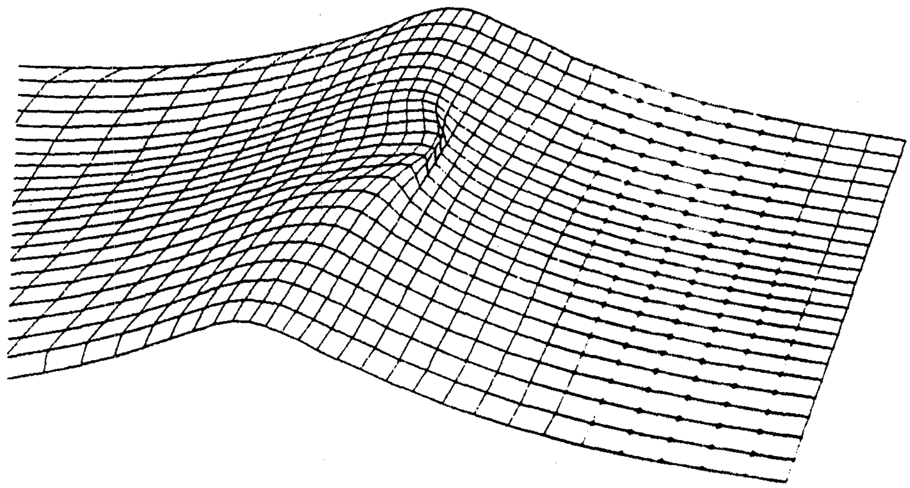

Fig.3. Detailed view of free surface grid shape during wave breaking.

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DISCUSSION

SCHULTZ: Your choice of even or odd splines is based on the length of time simulation of a breaking wave. Based on Dick Yue's discussion yesterday, it is possible that the instability you see in the odd splines is real and hence should not be suppressed. My breaking wave computations show that more accurate solutions often "fail" earlier.
Also your argument against the Runge-Kutta method seems superficial. It is obvious that a R-K technique will cost more per time step, however they are easily implemented by standard routines and could very easily be optimal in the sense of lowest computational cost at a fixed time discretization error.
BROEZE & ZANDBERGEN: The instabilities observed using splines of uneven degree are caused by the combination of ***unequal*** grid spacing with uneven spline degree. From an extensive analysis of this problem, we have concluded that the local Jacobian (from physical domain to computational domain) may become singular then. These singularities do not occur if more equal grid spacings are used, which proves that the phenomenon is numerical. Also it will not occur if splines of even degree are used.
A Runge-Kutta technique can be easily implemented, but implementing our 2-stage 2-derivative method involves only little additional effort per (intermediate) time level. That is why this method is much more inexpensive than the classical 4th order Runge-Kutta method.

TANIZAWA: Xü, H & Yue, D.K.P developed the Qubic panel method with 9 collocational points and showed the convergence test to the number of panels. To compare the convergence speed of your method, which is quadrilateral panels with one collocational point, with their method, please show the results of convergence test.
BROEZE & ZANDBERGEN: Results of convergence tests with our panel method have been presented by Romate in:
J.E. Romate: "The numerical simulation of nonlinear gravity waves in three dimensions using a higher order panel method", PhD thesis, Enschede, The Netherlands (1989).
J.E. Romate & P.J. Zandbergen: "Boundary integral equation formulations for free-surface flow problems in two and three dimensions", *Computational Mechanics*, Vol 4 (1989), pp 276-282.
His figures show that the method is third order convergent.

GREENHOW: Your domain moving scheme is similar to that of Vinje and Brevig, who apply periodicity there. You apply an inflow and outflow condition. How do you know these flows?
If you use the Rienecker & Fenton method, then periodicity is then implicit in the outflow and inflow.
BROEZE & ZANDBERGEN: At inflow boundaries, I prescribe the fluid velocity in normal direction. I prescribe the values on the basis of Fourier approximations obtained with Rienecker & Fenton's method, which are determined on the assumption of periodicity.

DISCUSSION

As a first test case, these data were used to check the accuracy and stability of our numerical method for highly nonlinear waves.
Using the values on inflow-boundaries, and radiating boundary conditions on outflow boundaries, we can calculate the deformation of waves (over bottom topologies or due to interaction with floating bodies) with our method.