NONLINEAR DYNAMICS OF A FLOATING BODY

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ABSTRACT

Recently there have been attempts to apply the study of dynamical systems (in particular their period-doubling and chaotic responses) to ship roll and capsize, and to other floating body problems such as the motion of a single-point mooring column. A good introduction is the book by Thompson and Stewart (1986). One can, of course question the validity of using any model based on the linearisation of the boundary-value problem for these type of large responses, but it is probably true to say that linearised hydrodynamics can be used to indicate the occurrence of dangerous or unusual motions, even if it cannot accurately predict their amplitudes. All of these studies have, however, used the usual added mass (m_a) and wave damping (b) concepts in the equation of motion, which itself may contain nonlinear restoring and damping terms. We here look at the equation

$$(m + m_a(\omega))\ddot{x} + \rho\pi a^2 C_D\dot{x}|\dot{x}| + b(\omega)\dot{x} + k_i x = F_{diff}$$

in which both m_a and b arise from the solution of the steady-state boundary-value problem of a body oscillating sinusoidally near a free surface with a frequency-dependent boundary condition. Hence the added-mass and damping concepts are not appropiate for any other type of body motion, including transience, see Maskell and Ursell (1970) for example.
Furthermore with the nonlinear terms damping and bilinear spring terms (k_i takes different values depending on the displacement direction), the steady-state motion will not be sinusoidal even in monochromatic steady waves. A further criticism of the use of the above equation is that the limit cycles of the motion sometimes require a very long time to establish themselves, which may mean that they would be difficult to realise even in a wave tank. Since the above equation cannot accurately predict the transience period, the engineering significance of these limit cycles is then open to question.
We have replaced the above equation of motion by (see Cummins (1962))

$$(m + m_a(\infty))\ddot{x} + \rho\pi a^2 C_D\dot{x}|\dot{x}| + k_i x + \int_0^t \kappa(t-\tau)\dot{x}(\tau)\,d\tau = F_{diff}$$

where the integral term accounts for the system memory which arises from previous body motion radiating waves; these consequently affect the present fluid motion and hence body forces. This memory is accounted for by the impulse response term which may be calculated as

$$\kappa(t-\tau) = \frac{2}{\pi}\int_0^\infty b(\omega)\cos[\omega(t-\tau)]\,d\omega$$

As can be see, the kernal depends on all frequencies, which could be problematical for general bodies. For the purposes of comparison of results from the two equations of motion, we take the simple case of a vertical cylinder moving in sway, for which the wave damping is known analytically for all frequencies. (An alternative would be to solve the time-domain boundary-value problem directly, see eg Newman(1985)). We plan to extend the study to isolated and interacting truncated cylinders at a later date.

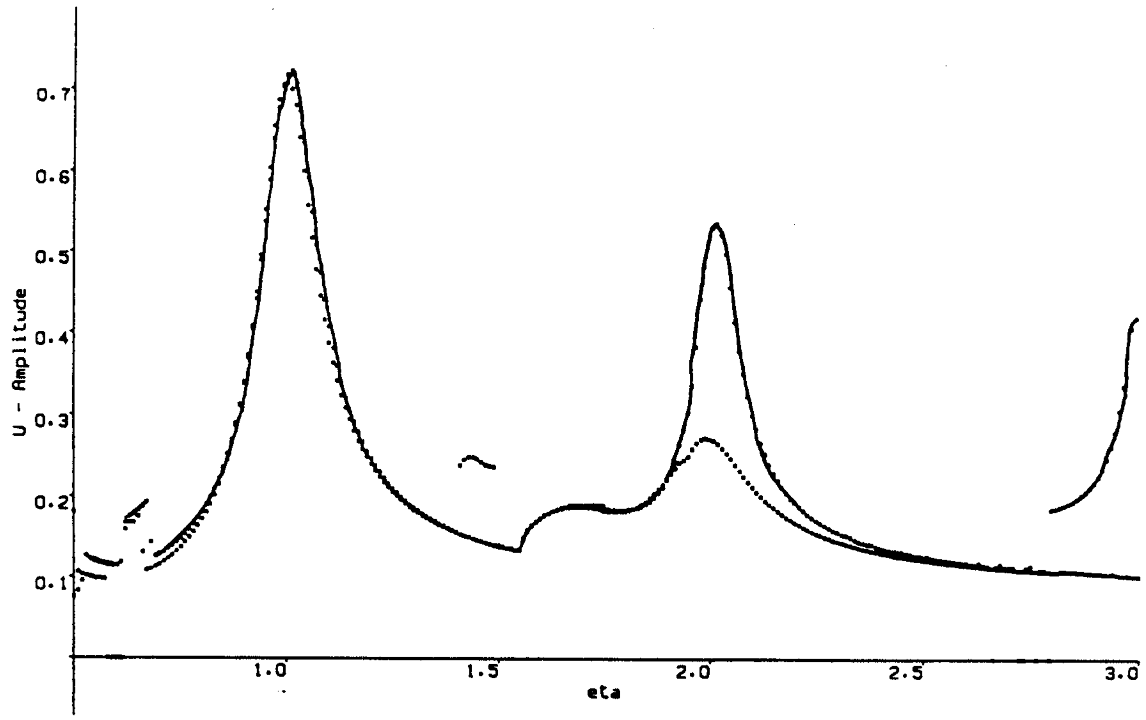
The figure shows an example of comparisons between the peak-to-peak velocities and the velocity coordinates of the Poincare points for the cylinder in waves of a fixed period, as predicted by the two equations of motion above (the solid/dotted line indicates results from the equation with/without memory). The system has viscous damping (C_D=0.7) and alpha = 10 specifies the ratio of the restoring spring constant in either direction. The axis - labelled eta - gives the ratio of the natural (bilinear) period to the forcing wave period. We see that the two results agree well for the position and response at eta=1, but are very different when the body responds with twice the wave period. This arises largely because we have chosen the wave damping in the mass-spring system as that at the forcing frequency (no other choice would be rational).

A fairly complete set of results for the parameter ranges have benn calculated and will be presented, as well as a comparison of the transients predicted by the two equations of motion.

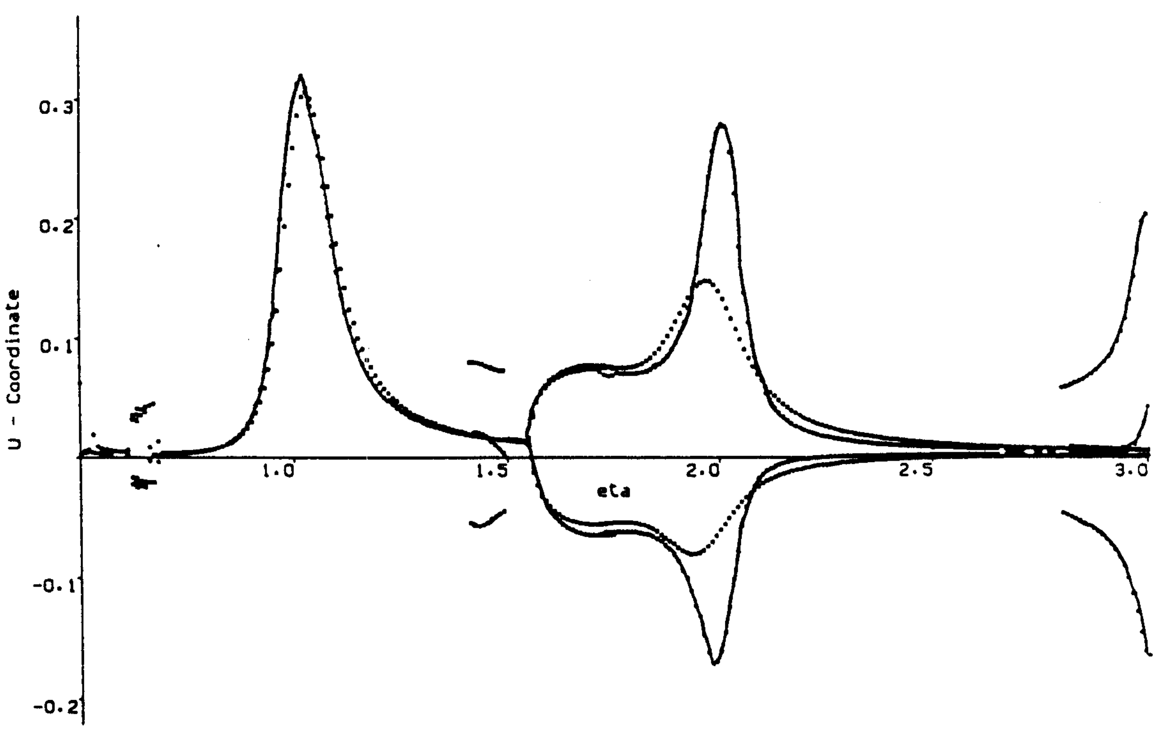
References

1) Cummins W E "The impulse response function and ship motions" Schifftechnik Bd 9 Heft 47, pp 101-109, (1962).
2) Maskell S J and Ursell F "The transient motion of a floating body" J Fluid Mech V 44 pt 2, pp 303-313, (1970).
3) Newman J N "Transient axisymmetric motion of a floating cylinder" J Fluid Mech V 157, pp 17-33 (1985).
4) Thompson J M T and Stewart H B " Nonlinear dynamics and chaos" see esp ch 14, John Wiley and Sons, (1986).

Peak to trough velocity v eta



Velocity of fixed points v eta



h/a= 5.0000 ka= 0.4083
Cd= 0.7000 A/a= 0.2000 alpha=10.0000
X(0)= 0.0000 U(0)= 0.0000

DISCUSSION

PAWLOWSKI: To justify somewhat practitioners in this area I would like to mention that they use added mass and damping coefficient evaluated at the dominant forcing frequency.
BENNETT & GREENHOW: Yes we do that too! However it is not possible to get the correct behavior over a wide range of applied springs with a 2^{nd} order ODE whatever you choose for the added mass and damping.

RAINEY: You have substantial reservations about the constant added mass models used by Michael Thomson — will this modify his most practically important conclusion, about the sudden erosion of the basin boundary (e.g. Rayney & Thomson, JSR, 1991)?
BENNETT & GREENHOW: We haven't looked at this yet. My feeling is that the sudden erosion will still occur, but almost certainly not at the same forcing amplitude. The situation for ship capsize may be easier to justify as a 2nd order ODE, since the damping is largely viscous, not wave damping, in roll, and hence may have less memory effect. For rig motions I believe memory to be vital for the accurate prediction of limit cycles and length of transience.