Wave diffraction by an array of cylinders in a channel

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Abstract

An array of vertical cylinders is placed in a channel in the presence of a regular incident wavetrain. The cylinders are assumed to be uniform and the waves lie within the linear wave regime. In the inviscid theory, the potential satisfies the Helmholtz equation in the fluid domain, together with the boundary conditions of zero normal derivative on the walls of the channel and cylinder.

The theory for a single circular cylinder in a channel is now well established. Spring and Monkmeyer (1975) considered a circular cylinder at the centre of the channel and demonstrated the influence of the channel walls by comparison of pressure and force results with those obtained by MacCamy and Fuchs (1954) for a single circular cylinder in the open sea. This work was extended by Eatock Taylor and Hung (1985) to include the case in which the cylinder could be placed in an offset position in the channel. Both of the channel papers cited above use the method of images. There is a troublesome slowly varying series is associated with this formulation and an efficient way of evaluating this numerically was given by Thomas (1987). This paper also presented the reflection and transmission coefficients, from which the mean horizontal in-line force on the structure can be determined.

For an array of circular cylinders in the open sea there is a substantial body of work. Important contributions have been made by Spring and Monkmeyer (1976) and Eatock Taylor (1985) using exact theories.

An interesting approximate theory, using the plane wave approximation, was given by McIver and Evans (1984) following the introduction of the concept by Simon (1982).

This paper considers an array of cylinders in a channel, with the aim of determining the influence of the channel walls by comparison to the open sea results in the same manner as previously done for the single cylinder. The solution uses the boundary element technique ; such a choice was made for two reasons. Firstly, it is not restricted to circular geometries even though it is readily acknowledged that such geometries are the main object of consideration. Secondly, the complexity of the Green's function formulation for an array of cylinders suggests that great computational advantage is not necessarily achieved by use of semi-analytical techniques.

Comparison of pressures and forces on the cylinders relative to their open sea values will be presented. Reflection and transmission coefficients will also be presented ; these have important influences on flume effects as well as enabling the mean horizontal in-line force on the array to be calculated.

This work is a joint venture with B.P. Butler (Numerical Algorithms Group, NPL, Teddington).

References

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DISCUSSION

Callan: For a spherical body in a canal, it is possible to recover the 'open-sea' potential by using the definition of the Riemann integral. This method probably works for your cylindrical potentials.

Thomas: This may well be true for a single cylinder in a channel. However, for an array of cylinders there is a much greater degree of difficulty; for this reason, a numerical approach was adopted.

Evans: You have shown how extra reflected waves occur as you widen the tank, for fixed incident wavelength. Clearly, the limit as tank width $\to \infty$ is non-uniform, so perhaps it is unrealistic to expect results for forces and pressures to approach the open-sea limit for a single cylinder?

Thomas: The limit is probably non-uniform, but there are two points which must be considered:

(i) the importance of the channel walls in experiments must be acknowledged; and
(ii) wall effects remain important as the channel width is increased to a surprising extent.

Both of these do not receive the practical consideration which they merit.

For a single cylinder, the pressures are considerably affected by the presence of channel walls, but the force does not vary much from the open-sea case. This behaviour does not hold for an array of cylinders: both pressures and forces differ considerably from their open-sea values. However, in both situations, there is evidence that the open-sea results will be recovered if the width is increased sufficiently. The non-uniformity means that the width must be very large, much larger than existing wave tanks, and so the point made by the discusser is valid for most physical wave experiments.

Tyvand: Consider two vertical circular cylinders in a channel; let α denote the angle between their line of centres and the channel walls. Your results for $\alpha = 0$ and $\alpha = \pi/2$ cannot represent the general case of oblique orientation ($0 < \alpha < \pi/2$). Will you find any new qualitative effects in the oblique case? Are such effects maximal (in some sense) when $\alpha \simeq \pi/4$?

Thomas: I doubt if any new effects will arise. In the absence of calculations, I would speculate that the greatest interaction occurs for in-line cylinders ($\alpha = 0$).

It seems unlikely to me that $\alpha = \pi/4$ will represent maximal interference in any sense. The definition of 'maximal' is difficult because interference depends upon frequencies, spacing, orientation and channel width; variations with these parameters are considerable and cannot be globally classified.

A simple conclusion would be: the presence of channel walls is important at all angles α, $0 \leq \alpha \leq \pi/2$.