SECOND-ORDER WAVE FORCES AND MOMENTS IN MULTIDIRECTIONAL SEAS

by

Jørgen R. Krokstad
Division of Marine Hydrodynamics
Norwegian Institute of Technology
N-7034 Trondheim-NTH, Norway
Email: KROKSTAD@sintef.no

Recent studies by Kim and Yue [1] indicate a pronounced effect on drift excitation forces when the interaction effect between incident wave components with different directional angle is taken into account. The major consequence of these investigations are that simple superposition of drift contributions from different incident wave angles in realistic sea conditions, may not be adequate for the prediction of slowly varying motions. The superposition principle must be considered as state of art in the hydrodynamic community. Kim and Yue's studies, among some surprisingly rear others, have motivated the author to further develop theoretical methods for computing the second-order directional interaction terms in a more extended hydrodynamic analyses.

The problem of directional interaction have been addressed by a stepwise methodical development:

1. An exact function representation of the mean drift forces and moment in monochromatic (single frequency) bidirectional waves on an elliptical shaped vertical cylinder. Infinite water depth is assumed. The effect of the complicated second-order wave velocity potential is without relevance for this formulation.
2. A hybrid panel formulation valid for low current velocities in bidirectional and consequently bichromatic waves, refer [2] for unidirectional waves. Here, we simplify by assuming single frequency of encounter induced by dual incident wave components on mean forces. Consistently, the effect of the second order wave potential can be neglected. Velocity dependent forces and moments in six degrees of freedom are computed. Hence, wave dependent damping terms are obtainable.
3. A general second-order force and moment formulation in bichromatic (different frequency) bidirectional waves on an elliptical shaped vertical cylinder. Finite water depth is assumed .The second order wave velocity potential is included by applying Green's theorem on a 3-D body. Second order quantities can be substituted by first order quantities . And finally, first order quantities can be superposed and interaction terms can be extracted.
4. Simulations and spectral methods in combinations with more exact statistical formulations are investigated. The main goal is to develop rational methods to estimate slowly varying responses in multidirectional seas within a reasonable CPU cost. Crucial wave conditions are shortcrested seas with arbitrary directional spread or unidirectional combined wave systems as storm and swell from different directions.

5. Experimental tests to validate computation of interaction terms on mean wave forces on a realistic floating ship hull. Most of the tests will be performed in bidirectional regular seas generated by running two wave generators simultaneously. In addition, it will be attempted two verify the importance of the interaction terms on the slowly vary response in irregular multidirectional seas by performing tests in shortcrested seas and cross seas with overlapping frequency regions.

The last three items will not be further commented since the considered methods are still under development.

One can argue that the above mentioned basic formulations even in single directional waves are not well enough established to justify further extension. This is why cross checks with even complicated exact solution are considered extremely important, especially for validation purposes.

In monochromatic bidirectional waves the mean drift force can be described by, refer Kim and Yue [1],

$$\overline{F_i} = A_1^2 D_{i_{11}} + A_1 A_2^* D_{i_{12}} + A_2 A_1^* D_{i_{21}} + A_2^2 D_{i_{22}} \qquad (1)$$

where $\overline{F_i}$ is the mean force or moment in freedom of degree no. i, A_1 and A_2 are the frequency and directional dependent complex wave amplitudes for incident wave direction no. 1 and no. 2 and $D_{i_{12}}$ is the corresponding quadratic transfer function. Note the symmetry relation $D_{i_{12}} = D_{i_{21}}^*$ The first and the last term in equation 1 represent superposed isolated drift force contributions, while the second and third terms represent nonlinear phase dependent interaction.

In addition to the drift forces in X and Y direction, as studied by Kim and Yue [1], it has been essential to study theoretical methods for the drift moment around the vertical Z-axis on directional sensitive bodies. A elliptical shaped cylinder was selected since exact solutions can be found in the X-Y plane. The elliptical eccentricity can be varied from a long ribbon to a circle.

The problem will be formulated in elliptical coordinates as defined in figure 1. The usual large volume body assumptions ideal fluid and incompressibility are applied. Laplace's equation and the ordinary boundary conditions on a fixed infinitively long cylinder can be substituted by elliptical coordinates after separation of Z- dependent depth functions. The 1. order diffracted velocity potential solution is expressed in terms of series representation of Mathieu functions of different type and order, refer Chen and Mei [3]. An asymptotic far field representation of the diffracted velocity potential is also applied. In addition to elliptical coordinates (ξ, η) and polar incident wave angle θ_I, see figure 1, the arguments to the Mathieu function solution will include the wave length versus interfocal distance ratio as expressed by the Mathieu parameter, $q = (kh/2)^2$, where k is wave number and h is interfocal distance defined by $h = \sqrt{(a^2 - b^2)}$. When the confocal ellipse attains a concentric circle, the diffracted velocity potential becomes the classical solution as worked out by MacCamy and Fuchs. Note that no frequency solution exists when $h = 0$. But it is possible to study solutions with a h value close to zero.

The quadratic transfer functions for drift forces and moment are obtained by integration the 1. order velocity potential cross terms expressed by Mathieu functions, up to second order in wave amplitude. The cross terms are obtained by substituting the total 1.order wave potential with $\varphi = A_1\varphi_1' + A_2\varphi_2'$. Two alternative methods are applied, a far field integration and a near field or direct pressure integration.

By using momentum conservation Newman [4] has derived expressions for the drift force moment in terms of the Kochin function or far-field velocity potential of the body. The Kochin function is directly related to the diffraction potential in the far field and can be substituted by the far field Mathieu function representation (Chen and Mei [3]). Ellipses tend to circles far away from the body so (ξ, η) are substituted by the polar (R, θ) coordinates in the Mathieu functions. With a consistent far-field formulation as Kim and Yue [1], a new quadratic transfer function expression for the drift moment around the vertical Z-axis has been derived.

The direct pressure integration method is based on substituting velocity potentials, X and Y dependent derivatives and integrals by elliptical coordinates. The mean second order force is found by taking the time average over one wave period. The effect of two directional wave components is superposed. Cross terms are extracted and integrated correctly to second order in amplitude on the wet surface. The resulting quadratic transfer functions for the drift force and moment are found. Here, we can illustrate the method by writing the expression for the finale quadratic transfer function of the drift moment, $D_{6_{12}}$,

$$\begin{aligned} D_{6_{12}} &= \frac{\rho g}{2k^2} \int_0^{2\pi} \frac{sin2\eta}{cosh2\xi_0 - cos2\eta} \frac{\partial g_1}{\partial \eta}(\xi_0, \eta) \frac{\partial g_2^*}{\partial \eta}(\xi_0, \eta) d\eta \\ &\quad - \frac{1}{4} \rho g h^2 \int_0^{2\pi} g_1(\xi_0, \eta) g_2^*(\xi_0, \eta) sin2\eta d\eta \end{aligned} \qquad (2)$$

where $g_1(\xi_0, \eta)$ is a Mathieu function representation of the velocity field from wave direction no. 1 at the wet body surface $\xi = \xi_0$. At this stage, no closed form solutions of the η dependent integrals have been found. The near field integrations are numerical more challenging than the far field integrals due to a more complicated and rapidly oscillating wave field close to the body surface. An adaptive quadrature algorithm was applied to ensure controllable accuracy. However, due to the analytic formulation, the far field and the near field method should give the same results except for inaccuracies in the involved numerical integrations. Numerical results for the far field solution were compared with the near field solution. When imposing an absolute error tolerance of 10^{-8} (double precision) the two methods corresponded with an accuracy of more than six decimals.

In table 1 examples of results for the quadratic transfer function are presented. The body considered is the the fixed infinitively long elliptical cylinder , as shown in fig. 2, with breadth length ratio 5:1 ($a = 0.5$, $b = 0.1$). By considering one frequency in the diffraction dominated region, $ka = 1.0$, and some few directions $\theta_I = 0, 45, 90$ (deg), a surprisingly large interaction effect can be illustrated. The results show that interaction effects on the moments are especially large for the wave directions 0 and 90 deg where no moment is induced by one of the two directional wave components isolated.

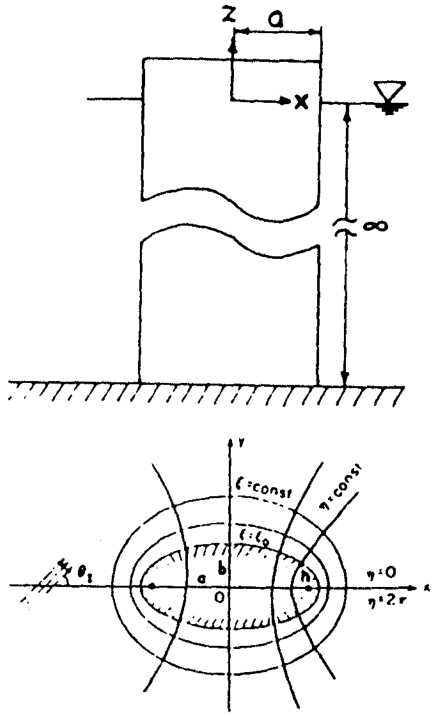

Figure 1.
Coordinate definition elliptical cylinder. The parameters a and b denote semi-major and semi-minor axis respectively. h denotes confocal distance.

θ_1 [deg]	θ_2 [deg]	$\frac{Re\{D_1\}}{\rho g a}$ $[N/m^2]$	$\frac{Im\{D_1\}}{\rho g a}$ $[N/m^2]$	$\frac{Re\{D_2\}}{\rho g a}$ $[N/m^2]$	$\frac{Im\{D_2\}}{\rho g a}$ $[N/m^2]$	$\frac{Re\{D_6\}}{\rho g a b}$ $[Nm/m^2]$	$\frac{Im\{D_6\}}{\rho g a b}$ $[Nm/m^2]$
0.0	0.0	0.0277	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	45.0	0.0250	-0.0037	0.0392	-0.0167	-1.3529	0.6751
0.0	90.0	0.0147	-0.0654	0.0583	0.0270	-1.9253	1.0283
45.0	45.0	0.1911	0.000	0.2641	0.0000	-2.0714	0.0000
45.0	90.0	0.1378	0.1786	0.4327	-0.0836	-1.4264	0.8994
90.0	90.0	0.0000	0.0000	0.7066	0.0000	0.0000	0.0000

Table 1.

Real part and imaginary part of the complex quadratic transfer function for drift forces and moment on a vertical elliptical cylinder with breadth length ratio 5:1 ($a = 0.5, b = 0.1$). Fixed wave length $ka = 1.0$ and different wave headings θ_I. $D_{1,2}$ denotes force in X and Y direction respectively and D_6 denotes moment around the Z-axis.

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