On the near field of a Kelvin wave source in the free surface, by F. Ursell.

Department of Mathematics, University of Manchester, M13 9PL, England.

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Abstract

The velocity potential of the Kelvin ship-wave is fundamental in the mathematical theory of the wave resistance of ships but is difficult to evaluate numerically. We shall be concerned with the integral term

$$F(x,\rho,\alpha) = \int_{-\infty}^{\infty} \exp\left\{-\tfrac{1}{2}\rho\cosh(2u-i\alpha)\right\} \cos(x\cosh u)\,du$$

in the source potential, where x and ρ are positive and $-\tfrac{1}{2}\pi < \alpha < \tfrac{1}{2}\pi$, which is difficult to evaluate when x and ρ are small. It will be shown here that

$$F(x,\rho,\alpha) = \tfrac{1}{2}f(x,\rho,\alpha) + \tfrac{1}{2}f(x,\rho,-\alpha) + \tfrac{1}{2}f(-x,\rho,\alpha) + \tfrac{1}{2}f(-x,\rho,-\alpha),$$

where $f(x,\rho,\alpha) = P_0(x,\rho e^{-i\alpha}) \sum g_m(x,\rho e^{i\alpha})\, c_m(x,\rho e^{-i\alpha})$

$+ P_1(\quad) \sum g_m(\quad)\, b_m(\quad)$

$+ \sum g_m(\quad)\, a_m(\quad)$.

In this expression each of the functions $g_m(\ )$, $a_m(\ )$, $b_m(\ )$, $c_m(\ )$, satisfies a simple three-term recurrence relation and tends rapidly to 0 for small x and ρ when $m \to \infty$, and the functions $P_0(\ )$ and $P_1(\ )$ are simply related to the parabolic cylinder functions of the single variable $-ix(2\rho)^{-\frac{1}{2}}\, e^{\frac{1}{2}i\alpha}$.