A SMALL FORWARD SPEED PERTURBATION METHOD FOR WAVE-BODY PROBLEMS

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The hydrodynamic behaviour of bodies at a small forward speed in waves have been studied by some authors; among those are Huijsmans and Hermans(1985), and Zhao and Faltinsen (1988). The translating pulsating source potential is used in all these investigations.

Here we propose an alternative method. The important features are that the forward speed effect can be separated from the zero forward speed solution and that we do not use the complicated pulsating translating source potential in numerical calculations. The advantage is that the forward speed effect can be taken into account by a post-processor to an existing diffraction-radiation computer program. Besides, in the range of forward speed where the theory is valid, the forward speed correction can be calculated once and for all.

First we shall consider some of the implications of relaxing the assumptions of slenderness. We define a steadily moving Cartesian coordinate system with the z axis pointing upwards, the x axis pointing in the direction of forward speed, and z = 0 chosen on the undisturbed water surface. At small forward speed, up to the first order of Froude number $F_n=U/\surd(gL)$, the free surface condition for the unsteady potential Φ is,

$$\Phi_{tt} + 2\vec{W}.\vec{\nabla}\Phi_t + g\Phi_z - U\bar{\phi}_{zz}\Phi_t = 0, \qquad \text{on } z=0, \tag{1}$$

where $\vec{W} = U\vec{\nabla}(\bar{\phi}-x)$, and $U\bar{\phi}$ is the steady potential which satisfies the "rigid wall" condition on the mean free surface. Thus it is given by the "double-body" solution. In the far field, the free surface condition reduces to

$$\Phi_{tt} - 2U\Phi_{xt} + g\Phi_z = 0, \qquad \text{on } z=0, \tag{2}$$

after the steady potential has been neglected. Note that this may not be justifiable if the steady potential does not satisfy the "rigid-wall" condition. These two conditions are used by Zhao and Faltinsen (1988). Within a linearized framework we can separate the time dependent factor by expressing the unsteady potential into a complex form $\Phi = \text{Re}[\phi e^{i\omega t}]$ with ω the frequency of oscillation. Suppose that all quantities can be expanded into Taylor series with respect to τ, for example,

$$\phi = \phi^{(0)} + \tau\phi^{(1)} + \ldots, \tag{3}$$

with $\tau = U\omega/g$. Substitution of this into the Laplace equation and boundary conditions leads to a zeroth order problem, a first order problem, and so on, after ordering in τ. The first two free surface conditions are

$$-k\phi^{(0)} + \phi_z^{(0)} = 0, \qquad \text{on } z=0, \tag{4}$$

$$-k\phi^{(1)} + \phi_z^{(1)} = -2i[\ (\bar{\phi}_x-1)\phi_x^{(0)} + \bar{\phi}_z\phi_z^{(0)}] + \bar{\phi}_{zz}\phi^{(0)}, \qquad \text{on } z=0. \tag{5}$$

In the far field equation (5) reduces to

$$-k\phi^{(1)} + \phi_z^{(1)} = 2i\phi_x^{(0)}, \qquad \text{on } z=0. \tag{6}$$

Now we assume that the water depth is infinite, and we divide the fluid region into an inner domain and an outer domain. For floating two dimensional bodies the two domains are separated by a large semicircle which encloses the body, with the centre on the free surface. For submerged bodies a circle which encloses the body is used.

In the inner domain various alternative numerical methods can be applied. In our implementation a boundary integral expression is used with the simplest Rankine source Green function. In the outer domain we use Green's second identity with a pulsating source Green function to express the potential ϕ by means of integrals over the joint surface S_J of the inner and outer domain, a control surface S_C at infinity and the free surface S_F bounded by S_J and S_C. After ordering, we have

$$\phi^{(0)} = \frac{1}{\alpha} \int_{S_J} \left(G\frac{\partial\phi^{(0)}}{\partial n} - \phi^{(0)}\frac{\partial G}{\partial n}\right) dS, \tag{7}$$

$$\phi^{(1)} = \frac{1}{\alpha} \int_{S_J} \left(G\frac{\partial\phi^{(1)}}{\partial n} - \phi^{(1)}\frac{\partial G}{\partial n}\right) dS + J, \tag{8}$$

where

$$J = \frac{1}{\alpha} \int_{S_F+S_C} \left(G\frac{\partial\phi^{(1)}}{\partial n} - \phi^{(1)}\frac{\partial G}{\partial n}\right) dS, \tag{9}$$

and for smooth body contours α takes the value 2π. The integral over S_C does not vanish because we have used a pulsating source Green function instead of a pulsating translating source Green function. The expression for $\phi^{(1)}$ is similar to that for $\phi^{(0)}$ except for an extra integral. To calculate this extra integral the asymptotic behaviour of the velocity potential is found to be useful. This can be obtained from the asymptotic behaviour of the potential corresponding to a translating pulsating source, given by Haskind (1954):

$$\tilde{G} \simeq -\frac{2\pi i}{\sqrt{1-4\tau^*}} e^{k(1+2\tau^*)[z+\eta-\mathrm{sgn}(x)i(x-\xi)]} + O(\tau^2), \qquad \text{for } |x| \to \infty, \tag{10}$$

where $\tau^* = \mathrm{sgn}(x)\tau$. The body velocity potential behaves similarly. With the asymptotic solutions we can integrate equation (9) explicitly over S_C. After some algebraic manipulations J reduces to

$$J = \frac{i}{\alpha}\left(2 \int_{S_F} G\frac{\partial\phi^{(0)}}{\partial x} dS - \left[(G\phi^{(0)})_{+\infty} - (G\phi^{(0)})_{-\infty}\right]\right) \tag{11}$$

where $\pm\infty$ indicate at $x=\pm\infty$, $z=0$. These three terms must be evaluated together in a limit sense.

So far we have not discussed whether the expansion (3) is valid throughout the fluid domain. In fact, such an expansion is only convergent in a local domain around the body. The divergence of the expansion in the far field is caused by the fact that the forward speed modifies not only the magnitude of the potential, but also the far field wavelength(cf. equation(10)). Therefore, strictly speaking, equation (9) is not appropriate. But equation (11) is valid and it can be derived on a rigorous basis. Suppose we decompose the total body potential into a far field solution plus a remaining term. The latter would be a local potential decaying to zero at large distances. The perturbation is therefore valid for this latter term throughout the fluid domain. For the integrals associated with the far field solution, integration can be performed analytically, and it can be shown that such integrals in the boundary integral expression can be expressed as functions evaluated at the intersection points of S_J with S_F. These functions can be expanded in τ since they are evaluated locally. Following this procedure, the result can be proven to be the same as equation (11). The perturbation theory is then complete.

The merit of the present method depends on how efficient the free surface integral J can be evaluated. Because the integrand is an oscillatory function, it is necessary to separate the oscillatory part of the integrand from the steadily decaying terms. It is desirable to find some analytical expression for $\phi^{(0)}$, and for this purpose we have developed a set of multipoles for the zero-speed solution such that the potential in the outer domain can be expressed in terms of multipole expansions. These multipoles differ from those obtained by Ursell(1949, 1950); they may be regarded as a generalization of those of Ursell(1949) for floating bodies, permitting floating and submerged bodies to be treated in the same way. Details will appear in a future paper. With these multipoles we can separate out the non-decaying oscillatory part of the integrand, which after explicit integration cancels the oscillatory parts of the last two terms in equation (11). The free surface integral can then be calculated effectively. Another difficulty for the problems of unsteady motion with forward speed is the evaluation of second derivatives of the steady flow potential in the body surface condition. This can be circumvented by using an integral transformation theorem developed by Ogilvie and Tuck(1969).

We have calculated the zero-speed solutions for several contours, using two numerical methods: (a) the inner domain integral expression coupled with the outer domain integral expression (7); (b) the inner domain integral expression coupled with the outer domain multipole expansions. Numerical results agree well with each other, and they also agree well with results obtained by other authors using different methods. They are not to be shown here, because of shortage of space.

We also have calculated the forward speed correction terms for a submerged circle, and some preliminary numerical results are plotted in Fig.1(a) and Fig.1(b). These are the variations of the sway-heave added mass and damping coefficients with frequency, for d/a=2, where d is the distance between the centre of the circle and the free surface and a is the radius of the circle. In the small forward speed theory, the cross-coupling coefficients are linearly proportional to τ. In these results only the far field free surface condition (6) is used, since the body is assumed to be deeply submerged. The numerical results agree well with those of Wu (1989) obtained from a more general method in which the forward speed is not assumed small. One should note, however, that the present small forward speed theory is expected to be invalid at very low frequencies, due to the simplification of the free surface condition. The strong oscillation of the numerical results as ka→0 suggests that they should be discarded.

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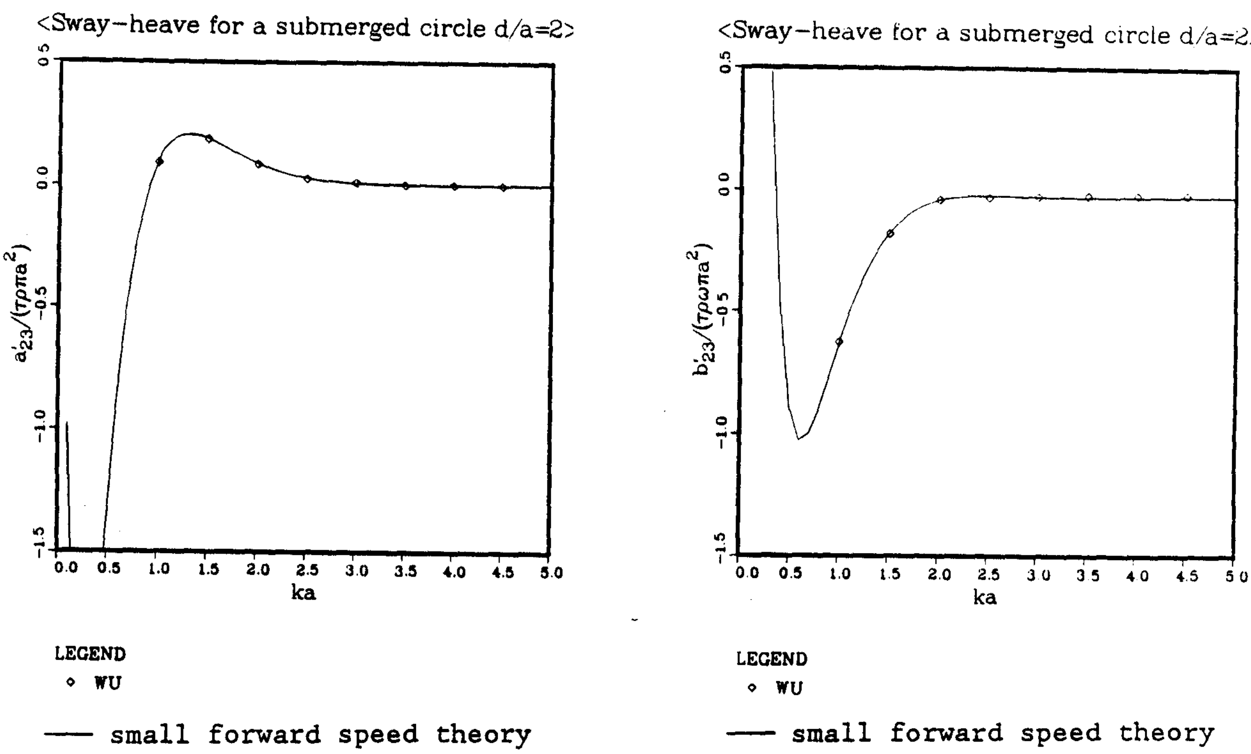

(a) Added mass (b) Radiation damping

Fig.1. Forward speed correction to the sway-heave coupling hydrodynamic coefficient for a submerged circle. d/a=2. (Zero-speed results: $a_{23}=b_{23}=0$)

DISCUSSION

Grue: Comment on the far-field expression of the 3-D Green function: There seems to be a misprint in the expression you quote from Haskind. This will cause a difference in the Kochin-function, say 10-20%, in the order of τ. Since the wave-drift damping is quadratic in the Kochin-function, this leads to a factor 20-40% in the wave-drift damping.

Hu & Eatock Taylor: We thank Dr. Grue for pointing this out, which is relevant to work on the three-dimensional problem. The results in the Abstract are for two dimensional bodies, and are therefore unaffected by the misprint in Haskind's paper.