On the decay of wave motion in water of finite depth

by

F. Ursell

Earlier work by Maskell and Ursell (JFM 44, 1970, 303-313) was concerned with the heaving of an immersed cylinder set in motion by an initial impulse or velocity on water of infinite depth; the corresponding problem for finite constant depth is now being studied by F.G. Robinson and involves many additional difficulties. A related problem is the Cauchy-Poisson problem for finite depth h: a surface impulse is applied near the origin, and the decay of the motion near the origin is studied. (There is no immersed body.) Suppose, in particular, that the impulse is proportional to $\exp(-x^2/4a^2)$ where a is a horizontal lengthscale, then the potential at time t at the origin is given by

$$I(t) = \int_0^\infty e^{-k^2a^2}\cos\{t\sqrt{gk \tanh kh}\}dk.$$

For large values of $T = t(g/h)^{\frac{1}{2}}$ this can be found by the method of steepest descents and is damped harmonic; for infinite depth, on the other hand, the motion decays algebraically when $t(g/a)^{\frac{1}{2}}$ is large. (This corresponds to the decay for a floating body, see JFM 19, 1964, 305-319.) Recently uniform asymptotics have been studied, valid for all large values of $t(g/a)^{\frac{1}{2}}$ and all small values of a/h; the leading term involves the integral

$$F_0(T) = \int_{\frac{1}{2}\pi i}^{\infty\exp(\frac{1}{6}\pi i)} \exp\{iT(z \tanh z)^{\frac{1}{2}}\}dz.$$

Discussion

Sclavounos: In Maskell's solution for the circular cylinder in infinite depth, the asymptotic behavior of the transient heave response for large time is an inverse power of time and is determined from the branch cut of the force coefficient at zero frequency. In finite depth the added mass is finite at zero frequency and consequently no branch cut is present. How does the corresponding heave response for large time compare to the infinite depth case?

Ursell: My result involves the combined variable $N\varepsilon^{1/2} = t(g/h)^{1/2}$. The infinite-depth result is obtained in the limit $N\varepsilon^{1/2} = 0$ and is algebraic, the finite-depth result is obtained in the limit $N\varepsilon^{1/2} \to \infty$ and is a complex exponential, i.e. damped harmonic. This latter result comes from the nearest saddle point; for finite depth the origin is a regular point.

Beck: Are there oscillations in the large-time asymptotics or does the curve approach zero smoothly?

Newman: There must be a direct connection between the $t \gg 1$ behavior and the $\omega \ll 1$ behavior in the frequency domain. Then the known results apply, with different results for 2D, 3D, infinite and finite-depth as discussed a few years ago by Yeung.

Yeung: My Applied Ocean Research paper of 1981 suggests that the 2D infinite-depth case has a low-frequency added mass and damping behavior similar to that of 3D finite-depth. If free-motion large-time behavior depends only on the limiting value of added mass and damping, it would explain the different behavior here. However, it may be more complicated than that.

Ursell: I should perhaps add that my result is obtained by expanding

$$e^{-\varepsilon^2 z^2} = \left[\sum_{m=0}^{\infty} \frac{(-1)^m}{m!} (z^2 - Z_1^2)^m \varepsilon^{2m}\right] e^{-\varepsilon^2 Z_1^2}$$

about the nearest saddle point Z_1, just as in the method of steepest descents (which is applicable when $N\varepsilon^{1/2}$ is large), but in my work the parameter $N\varepsilon^{1/2}$ may have any value. The details will be published elsewhere.