DIFFRACTION OF WATER WAVES BY A MOORED, HORIZONTAL, FLAT PLATE.

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The possibility of performing large scale, cost effective extraction of energy from the ocean waves has received considerable attention over the last two decades, motivated, no doubt, by the desire to obtain a clean, renewable energy source. In particular, a Norwegian research group has investigated the feasibility of constructing a system of underwater structures which would act like a lens and focus waves prior to harnessing their energy. Such a lens system would operate under the principles which govern the focussing of light waves. As a wave enters the shallower region over the submerged body, so the wave length is decreased and as is well-known, the wave speed is reduced. Thus, a phase lag is induced in the transmitted wave on the far side of the body. A water wave lens would be made up of several submerged bodies, each of which is capable of retarding the wave by a different amount.

Each lens element must clearly possess the property that it reflects very little of the incident wave over a wide range of frequencies. A notable candidate for such an element is a submerged, circular cylinder which is transparent to normally incident waves of all frequencies and the use of such a cylinder as a lens element has been reported elsewhere. In addition as the lens element would doubtless be moored in some way, it is necessary to determine how the reflection and transmission coefficients are affected by the motion of the element. It is also desirable, for cost purposes that the element should not be too bulky.

In the present work, linear theory is used to investigate the transmission of surface waves normally incident on a submerged, horizontal plate which is moored to the seabed by four, vertical, elastic cables, symmetrically placed around the plate. As a first approximation, it is assumed that two-dimensional motion only need be considered as if, for example, the plate were in a narrow wave tank. It is also assumed that the presence of the cables does not affect the wavefield. The usual assumptions of an inviscid, incompressible fluid and irrotational flow are made. Linear theory is used so the velocity potential may be split into two fundamental parts; one due to the scattering of waves by the fixed plate and the other due to the radiation of waves by the moving plate into otherwise calm water. The full solution to both the scattering and radiation potentials is obtained by the method of matched eigenfunction expansions, which is outlined below for the scattering potential. The equations of motion are derived which determine the amplitude of oscillation of the plate in each of its modes of motion.

The procedure of matched eigenfunctions is as follows. The symmetry of the geometry is exploited by splitting the potential into symmetric and antisymmetric parts which then need to be solved in the region $x > 0$ only. This area is then split into three regions; above the plate, below the plate and to the right of the plate. The potential is written as an eigenfunction expansion in each region, the coefficients of which are determined by requiring continuity of the potential and horizontal velocity on the boundaries of the regions. Systems of equations arise from the matching of the potential and horizontal velocity on the boundaries, which are split into real and imaginary parts, truncated at a suitable number of terms and solved using a standard Gaussian Elimination routine. It may be shown that, as a consequence of there being a square root singularity in the velocity at the ends of the plate, the coefficients in the potential expansion behave like $n^{-3/2}$ for large n.

Checks on the numerical results are provided by making comparisons with two approximate solutions. The first of these assumes that the plate width is large compared to the wavelength and the second is based on shallow water theory. Good agreement is obtained with the full results over the respective ranges of validity. The effect of the moorings on the response motion of the plate is examined and it is observed that whilst very stiff cables considerably reduce the motion of the plate from what it would be if it were neutrally buoyant, it is possible, by choosing an intermediate stiffness parameter, for the response motion to be larger than that for the neutrally buoyant plate over part of the frequency range. It is also observed that the heave response of the plate is zero when the plate width is an integral number of wavelengths, referring to the wavelength over the plate, independent of the choice of stiffness in the cables.

The effect of varying the mooring stiffness on the far field form of the potential is examined and it is shown that by an appropriate choice of the stiffness parameter, both the total reflection and total transmission coefficients are 180 degrees out of phase with what they would be for a fixed plate. Results are presented illustrating the variation of the amplitude of the total reflection coefficient and also the phase of the total transmission coefficient with frequency and it is found that a necessary condition to obtain a small reflection coefficient coupled with an appreciable phase change in the transmission coefficient, is that the plate width should be at least half to once times the incident wavelength.

Discussion

Beck: Why is there no heave-roll coupling?

McIver: There is no coupling if the moorings are vertical and symmetric.

T. Wu: Is flow separation important at the ends of the plate?

McIver: Yes, it is possible and Stiassnie has looked at vortex shedding.

Yeung: How does the convergence of the eigenseries relate to the square-root singularity at the end of the plate?

McIver: There is a square root singularity at the end of the plate, so the coefficients in the potential expansion go like $n^{-3/2}$.

Mehlum: Another possible approach is to use the Cauchy-integral approach which is very elegant and involves only say, 8 terms.

McIver: Yes, that is nice.

Ursell: How many parameters are there?

McIver: I have looked at a theoretical model. We did not say what experimental parameters we are looking at. We would have to look at thickness, etc.

Hearn: Regarding your conclusion that the motion response can be made to resonate by carefully selecting the mooring stiffness, is the converse true? In fact, the underlying idea behind tube pump moorings was to change the stiffness, to remove unwanted resonant motions. Did you treat explicitly the ratio of the hydrodynamic stiffness to the mooring stiffness or simply assign values of stiffness consistent with taut moorings?

McIver: I just assumed the moorings were taut.

Kleinman: Did Heins do the half plane problem with a linearized free surface condition, presumably using the Wiener-Hopf method?

McIver: Yes, you can actually get the reflection coefficient explicitly that way.

Newman: I am concerned about a highly tuned lens where the plate length is much longer than the wavelength. How can such a lens be useful over a broader spectrum of ocean waves?

McIver: Perhaps E. Mehlum can answer that.

Mehlum: You are quite right. However, there are body shapes which are more effective than the flat plate. For these shapes or contours I recommend a Cauchy formulation to do the calculations. The flat plate is interesting as a start to study these problems, but it is not very useful in practice for the reasons mentioned by Newman.