Higher Order Methods of Hydrodynamic Analysis

by

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Within the Hydromechanic's Research Group at Newcastle University, Higher Order Methods of Hydrodynamic Analysis are being investigated on two fronts.

Under numerical hydrodynamics a Higher Order Boundary Element (HOBE) method has been developed, implemented and tested for arbitrary 2D sections (floating or submerged) with free surface effects included. In developing the theory of higher order boundary elements both the radiation and diffraction problems have been investigated. Within the analysis program developed linear or curved elements may be used and on these elements the unknown dependent function can be represented by constant or higher orders of approximation (up to cubic used). Assessment of the technique has been carried out through comparisons with the conventional Frank close-fit approach in terms of cpu time required to formulate and solve the problems and the accuracy of the modelling. The Frank close-fit approach can be implemented using either the simple mid-point integrand evaluation approximation of integration over the facet or the more exact analytic integration. We have implemented both since the former method is usually satisfactory from a practical engineering point of view and the latter approach is nearer in complexity to the details of the HOBE method. The difference between the analytic integration approach and the implemented HOBE method provides some measure of the overheads associated with the new technique. One must also note that the reciprocity properties of the Green function cannot be exploited in the HOBE method. Our applications have confirmed that both accurate modelling and fewer higher order boundary elements are required to model the stated fluid-structure interactions. However, the full potential of the method can only be attained if significant time improvements can be made in the cpu time spent in formulating the stated interaction problems. Much of this additional demand for computer resources arises because of the management associated with free selection of N, the number of elements, n the order of approximation for the element geometry, m the order of approximation for the behaviour of the unknown on the element together with the significant amount of increased integration of the Green function and associated boundary condition values for the problems being investigated. Thus improved formulation times of the HOBE method will require additional work on identification of robust decision making rules to permit the computer code to automatically select "optimum" quadrature rules.

Within the theory the usual Fredholm Integral Equations have been reduced to equivalent algebraic problems and so formulation is essentially concerned with evaluation of the coefficients of the complex simultaneous sets of equations:

$$[\, A + i\, B \,]\, [\, \Phi_R + i\, \Phi_I \,] = [\, C + i\, D \,].$$

The coefficients of the matrices A,B, C and D are derived from terms of the form:

$$\int_{-1}^{1} M_k(t) \left[-g_j'(t) \frac{\partial G_R}{\partial \xi} + f_j'(t) \frac{\partial G_R}{\partial \eta} \right] dt,$$

$$\int_{-1}^{1} M_k(t) \left[-g_j'(t) \frac{\partial G_I}{\partial \xi} + f_j'(t) \frac{\partial G_I}{\partial \eta} \right] dt,$$

$$\sum_{j=1}^{N} \int_{-1}^{1} \left[G_R(p_i,q)\, V_{R_j} - G_I(p_i,q)\, V_{I_j} \right] dt$$

and

$$\sum_{j=1}^{N} \int_{-1}^{1} \left[G_R(p_i,q)\, V_{I_j} - G_I(p_i,q)\, V_{R_j} \right] dt$$

where

$$V_{R_j} = -g_j'(t)\, V_{x_R} + f_j'(t) V_{y_R}$$

and

$$V_{I_j} = -g_j'(t)\, V_{x_I} + f_j'(t) V_{y_I} \ .$$

The functions f(t) and g(t) represent the coordinates ξ and η of an element using the Lagrangian polynomial forms:

$$\xi = \sum_{i=1}^{n} \xi(q_i)\, N_i(t) = f(t)$$

and

$$\eta = \sum_{i=1}^{n} \eta(q_i)\, N_i(t) = g(t).$$

$N_i(t)$ are the usual shape functions of order (n-1). The functions $M_k(t)$, identical to $N_i(t)$, are used to represent the unknown velocity potential on the element in the form:

$$\Phi = \sum_{k=1}^{m} \Phi(q_k)\, M_k(t),$$

that is, a function of order (m-1).

When using a constant approximation for the unknown on an element the mid-point of the element is used to locate this value, whereas for higher orders of approximation a continuous solution arises since there is always a common unknown value at the meeting point of neighbouring elements. As a result of moving the singularities to the ends of the element in the higher order representations of Φ a number of interesting problems concerned with the singular nature of the associated Green function have arisen. These singularities are generally associated with the Rankine source part of the general Kelvin source used to model the free surface effects through the function G. For p_i a submerged point, boundedness of the integrations indicated above has been established for the various orders of approximation, n, used to represent the element geometry. Use of Lean and Wexler transformations also permits use of the Stroud and Secrest quadrature rule when evaluating the elements of the matrices C and D. For integrals free of any form of singularity Gauss-Legendre quadrature has been selected. Specification of the solid angle, α, nominally set to $-\pi$ in most 2D formulations, can significantly influence the solution near the free surface. We also find that the direct method of solution for the velocity potential is more readily implemented than the indirect or source strength approach.

The second front relates to calculation of the second order fluid damping coefficient by our Added Resistance Gradient Method. This is primarily concerned with evaluation of the gradients of the mean second order speed dependent forces with respect to the forward velocity as this velocity tends to zero. The theory is based upon the concept of low frequency wave damping outlined in Johan Wichers OTC papers of 1979, 1982 and 1984. This phenomenon is investigated using a generalised strip theory based method which was tested through comparisons between our theoretical predictions of the low frequency wave damping and NSMB's experimental measurements for a 220,000 dwt tanker. Because head sea diffraction is of interest Skjordal's generalisation of Maruo and Sasaki's head sea diffraction has been used to provide the wave excitation forces. In theory application of this analysis should provide a number of problems when looking at the Added Resistance Gradient as the forward speed tends to zero. However, once again the robustness of the ship theory unwittingly provides reasonable estimates of the sought second order wave damping coefficient. We would like to discuss some unresolved problems and our extension to a 3D analysis in the light of those problems concerning direct evaluation of source strength by means of the HOBE technique.

Discussion

Korsmeyer: I was pleased to hear your opening premise, that is, that we are interested in solving problems with large numbers of panels. Therefore, of course we wish to put our effort into the N^3 portion of the problem. What do you see as a practical value for a "large number of panels"?

Hearn: I have used 300 panels on a quarter body. For multi-bodies more panels are required.

X-J Wu: Have you compared computing times for conventional 3D and higher order panel 3D methods? I have tested linear and quadratic distributions over panels. The higher order panel method was first developed in aerodynamics. In that case, the Green's function is very simple, i.e., ln R in 2D cases or 1/r in 3D cases, such a technique is applicable. We, however, require more complicated Green's function evaluations and these can take much more computing time for evaluations using this method. Therefore, in 2D or 3D cases with a small number of panels, no savings in computer time can be expected. Obviously, as shown by Hearn and also by my own work, there is no improvement of the accuracy. In particular, when a structure has large curvature or more joints as frequently seen in the offshore industry, perhaps no substantial reduction of total panel number can be achieved. When a more complicated numerical technique can not gain either more accuracy or computing time saving (or both), its applicability may be doubtful. Nevertheless, in rigid body problems, the potential or source distribution over the structure midbody length varies slowly. Therefore, a suitable high order panel method in our field may be to apply linear, quadratic or cubic distribution over a large longitudinal strip, thus both numerical accuracy and savings in computer time can be achieved. This has been done in my work and given in my publication.

Breit: I have had some experience with higher-order panels in three dimensions. In my case, the geometry was represented exactly in the terms of orthogonal- curvilinear coordinates, however Chebyshev polynomials of varying orders were used to approximate the velocity potential. I found that piecewise linear variation of the unknown performed worse than piecewise-constant panels while piecewise-quadratic panels performed substantially better. This appears to confirm your experience in two dimensions.

Hearn: Piecewise-linear panels would seem to be less accurate than piecewise-constant panels.