Excitation of Three-dimensional Nonlinear Waves by Ships Moving in Shallow Water

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In recent years, there has been renewed interest in wave patterns generated by bodies propagating in shallow water. The original motivation comes from the recent experiments of Ertekin, Webster & Wehausen (1984) who observed that ships moving in channels at nearly critical speeds (Froude number based on depth about equal to one) continuously excite nonlinear waves which form and propagate ahead of the ship. In spite of the fact that the channel, where the experiments were conducted, was wide compared with the dimensions of the ship model, the observed nonlinear waves were, to a very good approximation, straight-crested solitons.

Using a two-dimensional travelling pressure distribution as an excitation, Wu & Wu (1982) solved numerically the Boussinesq equations and found solitons propagating ahead of the source. In independent work, Akylas (1984) demonstrated that, near critical conditions, the Boussinesq equations could be reduced to a forced Korteweg-deVries (KdV) equation; numerical solutions of this equation again show the appearance of solitons.

The three-dimensional aspects of the problem near critical conditions were first examined by Mei (1985) who showed that, for channels of finite but not very large width, the generated wave disturbance is again governed by a forced KdV equation. Also, Ertekin, Webster & Wehausen (private communication) solved numerically the forced three-dimensional Boussinesq equations in channels of finite, but not arbitrary, width and found two-dimensional solitons in front of the source, in agreement with their experiments.

This talk will focus attention on the wave disturbance generated by bodies moving in shallow channels of arbitrary (perhaps infinite) width, so that three-dimensional effects in the nonlinear waves found ahead of the source are not negligible as in the previous works cited above. It is shown that, near critical conditions, the generated wave disturbance is governed by a forced Kadomtsev-Petviashvilli (KP) equation, a natural generalization of the KdV equation found by Akylas (1984) in the two-dimensional problem. Of course, in the special case that the channel is not very wide, the three-dimensional effects drop out and the KdV is recovered, in accordance with Mei (1985).

The main question to be addressed is whether unsteady three-dimensional nonlinear waves appear in front of the moving body or a nonlinear steady state is reached. To answer this question, the predictions of the linearized theory, which is valid at small times, is examined first in an unbounded domain. It is found that the linearized response reaches a steady state which is oscillatory only behind the source. However, in front of the body, there is a finite disturbance, which varies like the inverse distance from the source and thus involves the displacement of an infinite amount of water. Also, a study of the transient linearized response reveals that, for large times, the linear dispersive effects decay faster than the nonlinear effects so that the nonlinear effects cannot be neglected in the far field. Thus, one is led to suspect that, as in the two-dimensional problem, nonlinear effects become important and give rise to three-dimensional unsteady nonlinear waves ahead of the body.

The nonlinear unsteady response is investigated by solving numerically the forced KP equation, using the predictions of the linear theory as initial conditions. The results of this numerical investigation will be presented and the effect of the channel width on the nature of the nonlinear wave disturbances found ahead of the source will be demonstrated. The transition from straight-crested solitons to three-dimensional nonlinear unsteady disturbances as the channel width is increased, will be discussed. Furthermore, the effect of reversing the sign of dispersion in the KP equation (which occurs at very small depth) on the soliton stability will be considered.

References

Akylas, T.R. (1984) J. Fluid Mech. 141, 455.
Ertekin, R.C., Webster, W.C. & Wehausen, J.V. (1984) In Proc. 15th Sympos. Naval Hydrodynamics, Hamburg.
Mei, C.C. (1985), submitted to J. Fluid Mech.
Wu, D.-M. & Wu, T.Y. (1982), In Proc. 14th Sympos. Naval Hydrodynamics, Ann Arbor.

Discussion

Wehausen: I would like to show three transparencies that complement Akylas' talk. All three show the waves generated by a rectangular pressure patch moving down a channel following an impulsive start. The first shows the waves for depth Froude numbers 0.9, 1.0 and 1.1 after a considerable time has elapsed. The generation of solitons ahead of the pressure patch is evident. Following the patch is the expected doubly corrugated surface. The next transparency shows for Froude number 1.2 how the solitons develop with passage of time. The last transparency is also for Froude number 1.2 but for pressure amplitude one third as high as in the last one. In this case, one sees that no solitons are generated. As a result of some two dimensional numerical experimentation it appears that for each Froude number > 1 there is a critical pressure amplitude beneath which no solitons are generated. I should like to add that the development of the "half-soliton" to a full one as shown in Akylas' computations resembles remarkably what we have observed in analogous situations.

Both T. Wu and I have computed and observed solitons for Froude numbers less than 0.4 (considered deep water). A theory must be able to predict these, and both the Boussinesq and Green-Naghdi equations do. This appears to be a shortcoming of Akylas' approach.

Akylas: I think the answer is that this goes back to what an asymptotic expansion is. The question is: How small is epsilon? However the perturbation theory usually extends far beyond its expected range of validity. Thus, I would not be surprised if the K-P equation predicts the appearance of solitons at such low Froude numbers. We plan to do such calculations in the near future.

T.Wu: I congratulate you on making some very interesting observations. There are now several models for this problem, so perhaps we should hold a small workshop in this area. Is it true that the K-P equation has a bias in favor of oblique solitons at small angles to the transverse direction? Based on limited computations, we have obtained results very similar to Akylas'. Regarding your comments about the group velocity; if we consider a single soliton where the phase velocity equals the square root of g(h + a), adding higher terms and computing the group velocity by differentiating would be a good way to find if we are on the right track for comparing the various theories. Finally, it is a "blessing of Mother Nature" that these theories have such a wide range of validity.

C. Lee: With the K-P forcing equation, can you use a bottom bump as well as a presure distribution to generate solitons?

Akylas: Yes, we can.

Mei: First, your abstract mentions computations for an elongated pressure distribution, but you have not presented these. Second, Wehausen and I disagree, within some parameter range, whether the waves are two-dimensional behind as well as ahead of the pressure disturbance. Third, you did not do any computations on cases similar to the one I studied. Did you say that you think the methods would agree?

Akylas: We have performed preliminary computations with both the linearized version and the nonlinear K-P in a channel of finite width. The nonlinear response shows the appearance of solitons. Unfortunately, I am not able to show the graphs today due to certain problems that we had with the laser printer. In the limit that the dimensionless channel width $d \ll 1$, where your theory applies, the response is two-dimensional. But for $d = O(1)$ this is not so behind the source.